Chapter 23
Magnetism

GOALS
When you have mastered the content of this chapter, you will be able to achieve the following goals:

Definitions
Define each of the following terms, and use it in an operational definition:
- magnetic field
- current sensitivity
- magnetic forces
- ferromagnetism
- ampere

Biot-Savart Law Apply the basic relationship between current and its associated magnetic field.

Magnetic Forces on Moving Particles Explain the motion of a charged particle in a uniform magnetic field.

Magnetic Interactions Discuss the interaction of magnetic fields.

Electric and Magnetic Fields Explain the difference between the behavior of charged particles in electric and magnetic fields.

Magnetic Field Applications Explain such applications as: electromagnetic pump, focusing of charged particles by a magnetic field, DC electric meters, motors, and the Hall effect.

PREREQUISITES
Before beginning this chapter you should have achieved the goals of Chapter 4, Forces and Newton’s Laws, Chapter 21, Electrical Properties of Matter, and Chapter 22, Basic Electrical Measurements.
Chapter 23
Magnetism

23.1 Introduction
Magnetism is a concept introduced in physics to help you understand one of the fundamental interactions in nature, the interaction between moving charges. Like the gravitational force and the electrostatic force, the magnetic force is an interaction-at-a-distance. Can you list five different ways in which magnetism has played a part in your life today? How strong is the earth's magnetic field? Are there interactions between living systems and the earth's magnetic field? In this chapter we will discuss the basis of the magnetic field model for the interaction between moving charges and explore the close relationship between electricity and magnetism.

23.2 The Magnetic Field Model
As early as 600 years B.C. it was known that naturally occurring lodestone would attract iron. This material was found in the country of Magnesia, and the name magnet was applied to individual specimens. It was found that when this material was suspended in such a way that it was free to rotate, it would align itself in an approximate north-south direction (Figure 23.1a). By studying the interaction between two specimens, it was found one pair of ends attracted each other and another pair of ends repelled each other (Figure 23.1b). The laws of interaction were similar to those of electrostatics. The relative strengths of the magnets were expressed in terms of pole strengths. The entire field of magnetostatics can be developed in a parallel manner to that for electrostatics. However, there is one great physical difference. To date there have been only a few disputed reports of detection of a magnetic monopole, which would be analogous to the single electric charge.

We shall approach our study of magnetism from the standpoint of magnetic effects of an electric current and the interaction between magnetic fields. We may say that in
any region in which a compass needle (a magnet) takes a definite direction, there exists a magnetic field. (If there were no magnetic field, the compass needle would take any random direction.) On the earth a compass takes a definite direction as a result of the earth's magnetic field; the earth itself behaves magnetically as a bar magnet. The compass needle aligns itself parallel to the magnetic field at the point of suspension. The end of the compass needle that points in a northerly direction is called the north pole, and the other end is called the south pole.

A dip needle is a magnetized needle that is mounted so that it can rotate freely in a vertical plane. When a dip needle is placed in a north-south plane, the needle points in the direction of the earth's magnetic field. The dip angle is measured from the horizontal position. For example, the dip angle at Washington, D.C., is 71°.

In 1873 James Clerk Maxwell published his famous theory of electricity and magnetism. In this work the magnetic field is ascribed to interactions involving moving charges. Moving charges are seen to be the sources of all magnetic phenomena. The magnetic field is introduced into physics to explain the interaction between moving charges, which is more complex than the Coulomb interaction between electric charges at rest. Maxwell's theory predicts that whenever there are moving charges, there are both electric and magnetic fields that can be used to explain observed physical phenomena. Another prediction of Maxwell's theory is that accelerating charges generate electromagnetic waves that travel with the speed of light.

23.3 Properties of the Magnetic Force
Assume that a magnetic field exists in the region of consideration and that there are no
electric or gravitational forces acting on the charged particles. We can explore the properties of the magnetic force, and hence the magnetic field, by directing a beam of charged particles into this region. The paths of the charged particles gives us information about the magnetic force $F_m$ which is acting upon them. The experimental facts are:

1. There exists one particular orientation in which $F_m$ is zero. This means that the charged particle goes in this direction at constant velocity. The line of the magnetic field, $B$, is this line, which is the same direction defined by the compass needle approach (Figure 23.2a).

2. For other orientations the magnetic force $F_m$ is always perpendicular to $v$ the velocity of the particle (Figure 23.2b).

3. The magnitude of the magnetic force is proportional to the charge $q$ and is in the opposite direction for positive and negative charges (Figure 23.2c).

4. The magnitude of the magnetic force is directly proportional to the component of the velocity perpendicular to the magnetic field; that is, $F_m \propto \sin \theta$ where $\theta$ is the angle between the velocity and the direction of the magnetic field $B$. From properties 3 and 4 we can define the magnitude of the magnetic induction field $B$:

$$B = \frac{F_m}{qv \sin \theta}$$

(23.1)
The SI units for $B$ are webers/m$^2$ (wb/m$^2$) or tesla(T) where 1 tesla = weber/m$^2$ = N/(C)(m/sec) 5. The magnetic force is always at right angles to the plane of the velocity line and line of the magnetic induction. Experiment shows that the direction relationship is as shown in Figure 23.3 where $v$ is the velocity of the positive particle. A right-hand rule determines the direction of magnetic force. The fingers point in the order of the multiplication (in this case $\sin \theta$ into $\sin \theta$), and the thumb will point in the direction of the product. The product is a vector and is perpendicular to the plane of $v$ and $B$. This is an operational procedure and definition, and is known as a vector product. It is normally written in the form

$$F_m = qv \times B = qvB\sin \theta$$

(23.2)

**EXAMPLES**

An electron of charge $-e$ (-1.60 x $10^{-19}$ C) is traveling east at 3.00 x $10^6$ m/sec in the magnetic field of the earth, 0.563 x $10^{-4}$ tesla (T) north. What is the direction and magnitude of the force on the electron?

We note the $v$ and $B$ are perpendicular to each other so $\sin \theta = 1$. Rotate fingers of your right hand from east to north and your thumb will point up, so the force on a positive charge would be up away from the earth, but an electron is negative so the force is directed down.

$$F_m = qvB = (1.60 \times 10^{-19} \text{ C})(3.00 \times 10^6 \text{ m/sec})(0.563 \times 10^{-4} \text{ T})$$

$$F_m = 2.70 \times 10^{-17} \text{ N}$$

2. A particle with a charge of 1.00 C is traveling with a velocity of
3.00 m/sec in a magnetic field of $7.00 \times 10^{-4}$ T. The particle experiences a force of $1.50 \times 10^{-3}$ N. What is the angle between the direction of motion and the magnetic field?

$$\sin \theta = \frac{F_m}{qvB} = \frac{1.50 \times 10^{-3}}{(1.00 \text{ C})(3.00 \text{ m/sec})(7.00 \times 10^{-4} \text{T})}$$

$$\sin \theta = \frac{1.50 \times 10^{-3}}{2.10 \times 10^{-3}} = 0.714 \quad \theta = 45.6^\circ$$

The quantity which we have been calling magnetic induction $B$, is also referred to as magnetic induction field, or magnetic flux density. The SI unit, weber/m$^2$ or tesla, is equal to $10^4$ gauss. The gauss (G) is the cgs unit for magnetic induction. To give you a feeling for the size of the units of magnetic fields, the magnetic field of the earth is of the order of $0.5 \times 10^{-4}$ Wb/m$^2$, a field of the order of 1 Wb/m$^2$ will pull steel rulers and screw drivers from pockets, and the maximum producible constant fields are of the order of 20 Wb/m$^2$.

### 23.4 Magnetic Effects of Electrical Currents

In this chapter we are interested in exploring the magnetic effects of moving charges. To show the magnetic effect of a current, which was first discovered in 1820 by Hans Christian Oersted, we can do the following experiment: place a wire carrying an electric current in the vicinity of a compass needle. Note the deflection of the needle both with current on and off. Reverse the direction of current, that is reverse the connections to the battery, and again note the deflections. We find that the compass needle is deflected by the presence of a magnetic field produced by the current in the conductor. The direction of the magnetic field at a point is defined as being in the direction of the force on a north magnetic pole at that point. The relationship between the direction of the magnetic field and of the direction of the electric current is given by a right-hand rule: If you place the fingers of your right hand around the current carrying conductor with your thumb pointing in the direction of the current (from positive terminal of battery to negative terminal in the external circuit), your fingers will encircle the conductor in the direction of the magnetic field. The direction of the magnetic induction for a long, straight conductor perpendicular to the page is shown in Figure 23.4.
The symbol $\otimes$ (inside a circle) represents a vector (direction of current) into the page. It may be considered the tail of an arrow. The symbol $\oplus$ (inside a circle) represents a vector (direction of current) out of this page. It may be considered the point of an arrow.

According to the Biot-Savart law, the magnitude of the magnetic induction resulting from a current in a circuit element $\Delta l$ at a point a distance and direction $\theta$ from $\Delta l$ is given by (see Figure 23.5):
\[ B = \frac{I}{2\pi r^2} (\Delta l \sin \theta) \]  
\text{(direction of } B \text{ given by the right-hand rule)} \tag{23.3} 

or in vector product form: 
\[ B = \frac{I}{2\pi r^2} (\Delta l \times \mathbf{r}) \]  
where \( \mathbf{r} \) is a unit vector.

One unit in length in the direction from the increment of length to the point of interest, \( \Delta l \) is the vector in the direction of the current flowing through the increment of the conductor of length \( \Delta l \), \( i \) is the current in the small element \( \Delta l \) (in the plane of the page) which establishes a magnetic induction contribution at point \( P \), and \( \mu_0 \) is the permeability constant. In SI units, \( i \) is in amperes, \( \Delta l \) is in meters, \( r \) is in meters, \( B \) is in webers per square meter, and \( \mu_0 \) has the value of \( 4\pi \times 10^{-7} \) weber per ampere-meter.

In order to develop the quantitative value of the magnetic induction due to a current in a conductor, one generally must use calculus methods. For special geometric shapes, we can find \( B \) directly from Equation 23.3. Let us apply Equation 23.3 to the case of the magnetic induction in the plane and at the center of a single circular turn of a conductor. For this case \( r = \) radius of the coil and is constant, \( \Delta l \) becomes \( 2\pi r \), and \( \sin \theta = 1 \). Thus
\[ B = \frac{\mu_0 I}{2r} \]  
\text{(23.4)}

For \( N \) turns, the value for \( B \) is \( N \) times larger. For a long straight conductor the value of \( B \) is given by:
\[ B = \frac{\mu_0 I}{2\pi r} \]  
\text{(23.5)}

(Figure 23.6a) where \( r \) is the distance from the conductor. Equation 23.4 and Equation 23.5 can be derived from Equation 23.3 by calculus methods (see Section 23.13). For a solenoid the value of \( B \) within the solenoid is given by:
\[ B = N\mu_0 \frac{I}{l} \]  
\text{(23.6)}

(See Figure 23.6) where \( N \) is the number of turns in solenoid and \( l \) is the length of the solenoid.
23.5 Motion of Charged Particles in a Uniform Magnetic Field

The fundamental relation for the magnetic force on a moving charge is given by Equation 23.2:

\[ \mathbf{F}_m = q \mathbf{v} \times \mathbf{B} \] (23.2)

As stated earlier, \( \mathbf{F}_m \) is always perpendicular to the velocity which means that there is no component of the force along the direction of motion, and no work is done by the magnetic force. With the velocity at right angles to the force we have the conditions for uniform circular motion (see Chapter 4). If a charged particle (charge = \( q \), mass = \( m \)) is projected with velocity \( \mathbf{v} \) at right angles to a uniform magnetic field \( \mathbf{B} \), the path of the particle will be a circle (Figure 23.7a). Because \( \sin \theta = 1 \), we then have:

\[ Bqv = mv^2/r \] (23.7)

and \( r = \frac{mv}{Bq} \) (23.8)

Thus we note that \( Bqr = mv = p \), the momentum of the particle; so the radius of the path of the particle is proportional to its momentum.

**EXAMPLE** A proton is projected with a velocity of \( 1.00 \times 10^6 \) m/sec perpendicular to a magnetic field of 0.100 Wb/m². What is the radius of its path?

\[ m_p = 1.67 \times 10^{-27} \text{ kg} \quad B = 0.100 \text{ Wb/m}^2 \quad q = 1.60 \times 10^{-19} \text{ C} \]

\[ r = \frac{1.67 \times 10^{-27} \times 10^6}{0.160 \times 10^{-19}} = 0.104 \text{ m} \]

If the charged particle, which is projected into a uniform magnetic field, has a component of velocity parallel to the direction of \( \mathbf{B} \), the particle will have a spiral path (Figure 23.7b) around the \( \mathbf{B} \) direction.
23.6 Applications of the Principles of Charged Particles Moving in a Magnetic Field

There are many applications involving charged particles moving in a magnetic field. Some important present-day instruments are electron microscopes, mass spectrometers, beta-ray spectrometers, cyclotrons, and betatrons.

Another application, which is important in some medical cases, is electromagnetic pumping. The basic electromagnetic pump is shown in Figure 23.8. A fluid which is a good conductor of electricity is placed in a closed system, circulated by means of an electromagnetic pump. A magnetic field is established over a portion of the closed system, and an electric current is passed through the fluid at right angles to the magnetic field. The portion of the fluid carrying the current then experiences a force whose direction is given by the right-hand rule. The fluid moves through the system as each element of fluid reaches this section of the system and experiences this force. The fluid moves through the system as long as the magnetic field and the electric current are maintained. The electromagnetic pump has been used to circulate blood in an artificial heart machine. An external magnetic field is created and the ions in the blood carry the electric current. Thus the pumping action can be achieved. The electromagnetic pump is preferable to the mechanical pump because a mechanical pump has moving parts that may damage the blood cells, but the electromagnetic pump has no moving parts and does not injure the blood cells.

23.7 Interaction Between Magnetic Fields

There is a simple experiment that we can do to show the interaction of magnetic fields:

We place a conductor between the poles of a strong magnet perpendicular to the field. A current is sent through the conductor, and we notice that there is a force acting upon the conductor when it carries a current and that there is no force when there is no current. We also notice that the direction of the force depends upon the direction of the magnetic field and the direction of current in the conductor.

The interaction of magnetic fields is shown in Figure 23.9.

**FIGURE 23.9**
A right-hand rule is applied to give the direction of the force on a current at right angles to an external magnetic field.

Where \( B \) is the direction of the magnetic field due to the magnet, the current in the conductor is up in the figure, and the direction of the force is out of the page. Let us look at the example in Figure 23.9 in terms of interacting magnetic fields. The magnetic field produced by the current in the conductor is clockwise when looking in the direction of the current. In back of the conductor the current \( B \) field is in the same direction as applied \( B \), and in front of the conductor the field from the current is in the opposite direction to the applied \( B \). We can
say that the total field is increased behind the conductor and decreased in front of it, and the direction of the force is from the stronger to the weaker field. This may be shown by use of a right-hand rule by directing the thumb, first finger, and middle finger so that they are mutually perpendicular to each other with the thumb representing direction of force, the first finger, the direction of current, and the middle finger the direction of magnetic field of the magnet (Figure 23.9).

Because a magnetic field exerts a force on a moving charge, we expect it to exert a force on the moving charges constituting a current in a conductor. Consider the current to be carried by free electrons which have a drift speed \( v_d \) to the left Figure 23.10. This means the conventional current is to the right, with the magnetic field into the page. The direction of the magnetic force is toward the top of the page. The current carrying conductor experiences an upward force.

**FIGURE 23.10**
The magnetic force on electrons moving in a magnetic field. Note that this is an example of a left-hand rule for negative charge.

### 23.8 Measuring Magnetic Fields

Let us develop a quantitative expression for the magnetic force on a wire. We have learned that the magnitude of the force on a charged particle is: \( F = qvB \sin \theta \). The force on a length \( L \) is then said to be the product of the number of particles in length \( L \) and the force on each. The number of free charged particles is the number per unit volume times the volume, that is, \( nAL \), where \( n \) is the number of conduction particles per unit volume, \( A \) is the cross-sectional area of the conductor, and \( L \) is the length. Then, the total force magnitude is just \( nAL \) times the force on an individual charged particle, \( F_m = nALqvB \sin \theta \).

(23.9) but the total current is given by the product of the number of charge carriers per unit volume times their charge, their speed, and the cross-sectional area of the conductor, \( I = nALqv \) 

(23.10) If the wire is perpendicular to the magnetic field, the force becomes: \( F = BIL \) 

(23.11) One newton is the magnetic force on a 1 meter conductor, perpendicular to a magnetic induction of 1 Wb/m\(^2\) and carrying a current of 1 A. By combining Equations 23.5 and 23.11 one can arrive at the following definition of the ampere:

One ampere is that unvarying current, which, if present in each of two parallel conductors of infinite length, and one meter apart in empty space, causes each conductor to experience a force of exactly 2x \( 10^{-7} \) newtons per meter of length.
23.9 Force and Torque on a Loop Conductor

In many basic galvanometers a rectangular loop (width \(b\), length \(a\)) of wire is suspended in a uniform magnetic induction. If a line perpendicular to the plane of the coil makes an angle \(\theta\) with the field \(B\) when current passes through the coil (see Figure 23.11a and 23.11b), there is a force upon each side of the coil in accordance with Equation 23.11. However, the forces on side \(AB\) and side \(CD\) cancel. The forces upon sides \(BC\) and \(DA\) are oppositely directed and exert a torque on the coil. The magnitude of the force is \(F_m = BIlb\). For \(N\) turns the force is \(N\) times as large.

**FIGURE 23.11**
(a) The magnetic force on a current carrying loop in a uniform magnetic field. (b) A side view of the loop in the magnetic field.
The torque about the axis of suspension is given by the force times the perpendicular distances
\[
\tau = 2F_mD = 2NBbasin\theta/2 = NBIAsin\theta \quad (23.12)
\]
for a coil of \( N \) turns, and where the area of the coil \( A \) is equal to \( ab \). In the case of a galvanometer, ammeter, or voltmeter the deflecting torque given above is opposed by a restoring torque which depends upon the suspending system. This restoring torque is of the form
\[
\tau_r = k\phi \quad (23.13)
\]
where \( \phi \) is the angle of deflection in radians and \( k \) is a constant of the suspension system. The coil takes an equilibrium position in which the torque produced by the current in the magnetic induction is equal to the restoring torque: \( BIA \sin \theta = k\phi \) (23.14) In many instruments the construction is such that the magnetic field always points in a radial direction, that is, \( \theta = 90^\circ \). Then (see Figure 23.12)

![Figure 23.12](image)

Equation 23.14 reduces to,
\[
I = (k/NBA)\phi \quad (23.15)
\]
where \( N \) = number of turns. For a given system \( k/NBA \) is a constant, and one has \( I = k\phi \) (23.16) where \( K \) is called the current sensitivity of the instrument.

The basic principle of an electric motor is the same as that of the loop in a magnetic field except the motor is constructed so that the torque is in the same direction at all times. With a torque applied to a coil or armature in a constant direction, rotation is produced. A rotating body has the ability to do work. We can convert electrical energy into mechanical energy. We can define any device that has an input of electrical energy and an output of mechanical energy as an electric motor.

**23.10 The Hall Effect**

One application of the forces on the moving charges in a conductor in a magnetic field is in the Hall effect. In this application a flat conductor is placed perpendicular to a magnetic field. If we consider the moving charge to be positive, then the diagram is as shown in Figure 23.13.
We expect an excess of negative charges on the back edge and an excess of positive charge on the front edge. This charge separation results in an electric force in the conductor. At equilibrium, the force resulting from the magnetic field is equal to the electric force due to the charge separation. This separation of charge produces an electric potential difference between the edges which is called the Hall voltage. In Figure 23.13 we would expect the front to be higher potential. If the moving charge carriers are negative, then the front of the conductor will be negative or at a lower potential. Hence the sign of the Hall potential indicates the sign of the charge carriers. Let us derive an expression for the magnitude of the Hall potential. At equilibrium the electric force magnitude will be equal to the magnetic force magnitude, \( F_e = F_m \), where the electric force is given by \( qE_H \), the magnitude of the charge on the moving charges, times the electric field \( E_H \) for Hall field, and where the magnetic force magnitude is given by \( Bqv_d \) where \( v_d \) is the drift speed of the charges in the conductors. (This magnetic force is in the opposite direction from the electric force.)

\[
qE_H = Bqv_d \quad (23.17)
\]

If the width of the strip is \( w \), then \( E_H = \frac{V_H}{w} \) where \( V_H \) is the Hall voltage across the strip, and we can write the expression for the Hall voltage in terms of the magnetic field \( V_H = BwI/nqd \) \( (23.18) \) But the drift velocity of the charges is difficult to measure; so let us recall that the total current \( I \) is given by the product of the number of charge carriers per unit volume \( n \) times their charges times their velocity \( v_d \) times the area of the conductor \( A \): \( I = nqvdA \) where \( A \) is equal to the product of \( w \) and \( d \), that is, the thickness in the direction of \( B \). We can substitute this expression in Equation 23.18 to eliminate \( v_d \), \( V_H = BwI/nqd \) \( (23.19) \). In the above equation all quantities except \( n \) can be measured in the laboratory, enabling us to calculate the value of \( n \). For monovalent metallic conductors the value of \( n \) is nearly the same as the atom density. Once \( n \) is known for a given material, then a Hall effect probe can be used to measure magnetic fields.
23.11 Electron Microscope

Earlier you learned that the path of a charged particle could be altered by application of either an electric field or a magnetic field. In Chapter 18 you studied an optical microscope. A microscope which uses a beam of electrons instead of a ray of light is called an electron microscope. The comparison of the two are shown in Figure 23.14.

In the electron microscope the lenses for focusing the beam of electrons may be either electrostatic or magnetic. In the United States the magnetic focusing lenses are more commonly used. Although there are some parallel features between optical and electron microscopes, there are also significant differences. Some of these differences include:

1. The magnetic lens of an electron microscope does not have a fixed focal length as does an optical lens. The focal length of the magnetic lens depends upon the strength of the field, which is altered by changes in the current in the coil.

2. In an electron microscope the magnification is not changed by using different lenses, as is the case in the optical unit, but by changing the focal lengths.

3. The depth of field in the optical microscope can be observed directly by altering the focus control. The visual observation in electron microscopes is not so acute.

4. The image formation is different in the two systems. In the optical unit image formation results from a differential absorption of the light incident by the specimens. For the electron microscope variations in intensity of the image depends upon the scattering of the electrons by the individual elements of the specimen. One great advantage of the electron microscope over an optical unit is the much greater magnification and higher resolving power. Some electron microscopes have a magnification of 200,000 and are capable of separating objects of a few angstrom units ($10^{-10}$ m) in size.

There were two difficulties with the electron microscope: only dead specimens could be used, and the images formed were entirely two-dimensional. The first difficulty results from the low penetrating power of the electrons. The thickness of water in a living cell is a great barrier to the electrons, and impairs the image. In order to study structures in three dimensions, generally a large number of sections were required. This meant each section must be photographed, and from a
number of photographs a three-dimensional model of the object constructed. However, the recent development of the scanning electron microscope has allowed research scientists to obtain some striking three-dimensional photographs of a variety of specimens. In biological work one is interested in obtaining contrast in the microscope image. The difficulties outlined above make this difficult in electron microscopes. Nevertheless, the electron microscope is an important instrument for life scientists and the medical profession. If you wish to study more fully the contributions of the electron microscope to various areas, you should consult a bibliography of electron microscopy. The literature contains many references and observations about the structure of bacteria, cells, tissues, and viruses. The optical microscope was developed over a period of about two centuries. The first commercial electron microscope was completed less than fifty years ago. Much progress has been made since that time, and as the recent development of the scanning electron microscope indicates, we can anticipate more improvements in the future.

23.12 The Mass Spectrometer

The mass spectrometer is an instrument designed to measure and compare masses of individual atoms. The atoms to be analyzed are first ionized and passed through an accelerator and a velocity selector before they enter a uniform magnetic field that bends them into circular paths. A schematic diagram of a mass spectrometer is shown in Figure 23.15. All of the particles enter into the main chamber with the same velocity.

The magnetic field \( B \) in this chamber is directed perpendicular to the plane of the path of the particles. The equation for the motion of the particles was discussed in Section 23.5, where we found the radius of the circular path of the particles to be proportional to their momentum \( mv \),

\[
r = \frac{mv}{qB}
\]

(23.8) If all the particles have the same velocity, particles of different ratios of mass to charge \( m/q \) are detected at different radii in the mass spectrometer.

23.13 Magnetic Materials and Biomagnetism

Since the discovery of magnetic monopoles (point sources for magnetic fields) has not been completely verified, we may assume that the sources of magnetic fields inside materials are atomic current loops (sometimes called magnetic dipoles). Materials are classified as
ferromagnetic, paramagnetic, or diamagnetic depending on the nature of the atomic current loops within the material. Ferromagnetic materials (iron, nickel, and cobalt at room temperature) have permanent atomic magnetic dipoles that align with external magnetic fields. This cooperative interaction very strong internal magnetic fields within ferromagnetic materials.

One model for ferromagnetism is based on electron current loops in the atoms of these elements. The electronic magnetic dipoles combine to form macroscopic magnetic regions (domains) that align with external magnetic fields to produce unusually large magnetization. Ferromagnetic materials and their alloys are the basic materials for our permanent magnets and electromagnet cores. The magnetic induction of magnetic material inside a solenoid is determined by its permeability $K_m B = K_m \mu_0 NI/l$ so materials of large permeability, such as iron with a $K_m$ ranging from a few hundred to ten thousand can greatly increase the magnetic field strength of a solenoid. You can make an electromagnet by using a piece of soft iron, a coil of wire, and a dry cell (Figure 23.16). The physical basis for this electromagnet is as follows. The external field due to the current in the coil interacts with the electronic magnetic dipoles in the domains of the iron. This interaction causes the domains to align their magnetic fields with the direction of the external field. This alignment process continues with increasing external field until all the domains are aligned and saturation is reached. The iron is then said to be completely magnetized. If the current is turned off, the domains remain aligned giving the iron a permanent magnetic field. This magnetic field can be reduced by heating the iron (allowing the domains to randomize) or by demagnetizing the iron with a field in the opposite direction. Ferromagnetism involves an interesting application of quantum physics and electromagnetism.

Paramagnetic materials such as liquid oxygen have molecular magnetic dipoles which tend to align with external magnetic fields. Paramagnetism is much weaker than ferromagnetism, but it can be used to study molecular electronic structure, as well as reaction rates for certain biochemical processes (enzyme catalyzed reactions, for example). Most organic molecules are diamagnetic, that is they have no permanent magnetic dipoles. When they are placed in a magnetic field, atomic current loops are set up that produce magnetic fields that oppose the external field. In biological materials we need only to note that one place we might expect to find significant magnetic interactions is in systems that include ferromagnetic atoms. Recent research suggests that birds are capable of using the earth’s magnetic field for navigation. The basis for such a magnetic sense is not yet known. The entire area of biomagnetism still lacks firm cases of magnetic phenomena. Recent sophisticated methods and instrumentation promise more insight into possible biomagnetic phenomena.
Magnetic fields of the human body. (a) The magnetic field of a human subject's heart is measured by a superconducting magnetometer inside the MIT shielded room. The shielding is provided by a five-layered wall with the addition of negative-feedback loops. The magnetometer is capable of detecting fields of the nanogauss range ($10^{-10}$ tesla). Fields produced by a normal human heart are of the microgauss range ($10^{-6}$ tesla), in accord with table shown. (b) Levels of the fields around the body and in the background. The strongest fluctuating field is over the lower part of the heart; the strongest steady fields, in welders and asbestos miners, are produced by particles of iron oxide in the lungs. The bottom of the diagram indicates the sensitivity of the two detectors, fluxgate and SQUID, for a 1 Hz bandwidth; the broken line shows that the limit moves to the right with greater bandwidth.

Details were published in an article by David Cohen in *Physics Today*, August 1975, page 34. (Courtesy of Dr. David Cohen.)
ENRICHMENT

23.14 Calculus Derivations Using the Biot-Savart Law

We will make use of the Biot-Savart Law, Equation 23.3, to calculate the magnetic induction of a long, straight conductor at point \( P \) (see Figure 23.17).

\[
|dB| = \left( \frac{\mu_0}{4\pi} \right) \left( \frac{Idx}{s^2} \right) \sin \theta \tag{23.9}
\]

The induction \( dB \) set up at \( P \) by any other element of the conductor is parallel to the vector shown. The resultant is the algebraic sum or the integral of the \( dB \). It becomes simpler if we let \( \theta \) be the independent variable,

\[
s = r \csc \theta x = r \cot \theta dx = -r \csc^2 \theta \, d\theta \text{Substituting, } dB = - \left( \frac{\mu_0}{4\pi} \right) \left( \frac{1}{r} \right) \sin \theta \, d\theta \, dB = - \left( \frac{\mu_0}{4\pi} \right) \left( \frac{1}{r} \right) \int_0^\pi \sin \theta \, d\theta = \left( \frac{\mu_0}{4\pi} \right) \left( \frac{1}{r} \right) [\cos \theta]_0^\pi = \left( \frac{\mu_0}{4\pi} \right) \left( \frac{2I}{r} \right) = \mu_0 I / (2\pi r) \tag{23.5}
\]

We will now apply the Biot-Savart law to calculate the magnetic induction of current in a single turn of wire (Figure 23.18).

Many devices use multiples of this configuration. A coil of one turn with radius \( R \) is in the \( yz \) plane with center at the origin. The increment \( dB \) is resolved into two components \( dB_\perp \) and \( dB_\parallel \). For a point on the x-axis all \( dB_\perp \) cancel out, and \( B \) total is the sum of \( dB_\parallel \) contributions.
\[ dB_\parallel = dB \sin \alpha \]

\[ dB_\parallel = \frac{\mu_0}{4\pi} I \frac{s \sin \alpha}{s^2} \sin \theta \]

As \( \sin \theta = \sin 90^\circ = 1 \),

\[ s^2 = R^2 + x^2 \]

\[ dl = R \, d\phi \]

\[ \sin \alpha = \sqrt{R^2 + x^2} \]

\[ B = \int dB = \frac{\mu_0}{4\pi} I \int_0^{2\pi} \frac{R}{(R^2 + x^2)^{3/2}} R \, d\phi \]

\[ B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}} \]

Note that if \( x = 0 \), this reduces to

\[ B = \frac{\mu_0}{2R} I \]

**SUMMARY**

Use these questions to evaluate how well you have achieved the goals of this chapter. The answers to these questions are given at the end of this summary with the number of the section where you can find related content material.

**Definitions**

1. The ampere is defined in terms of the force between two parallel wires; it is the specified
   a. force/meter
   b. current
   c. separation of wires
   d. area of each wire

2. The magnetic field is introduced in physics because it
   a. is analogous to the electric field in all interactions
   b. is analogous to the gravitational field in all interactions
   c. can be used in explaining interactions of currents
   d. was defined in the Bible
   e. explains the origin of magnetic monopoles
3. The force between parallel currents in the same direction is a. repulsive  
   b. zero  
   c. parallel to current  
   d. attractive  
   e. none of these  
4. The deflection of a galvanometer coil is due to a. suspension system heating  
   b. gravitational attraction due to current  
   c. diamagnetism  
   d. paramagnetism  
   e. torque on a loop conductor in a **B** field  
5. Ferromagnetism is based on atomic current loops. It is characterized by internal  
   magnetic fields that are: a. weak  
   b. nearly zero  
   c. independent of external fields  
   d. dependent on external fields  
   e. strong **Biot-Savart Law**6. For a current flowing into the paper **(□)**, its magnetic  
   
   ![Diagram](image)

   field will be directed as shown.  
7. For a long straight wire the magnetic field varies with the distance **r** from the wire  
   and it is proportional to a. **rb**.  
   b. **1/r**c.  
   d. **r**^2d. **SQR RT** (**r**)  
   e. **1/r**  
8. **Magnetic Forces on Moving Particles**8. The direction of the force on a charged  
   particle moving with velocity **v** in a magnetic field **B** is a. perpendicular to **v** but not  
   to **Bb. parallel to **v** but not to **Bc. perpendicular to **v** and **Bd. parallel to **B** but not to **v**  
   e. in the plane of **v** and **B**  
9. The magnetic force on a charged particle moving through a uniform magnetic field **B**  
   always produces a. no change in energy of the particle  
   b. no change in direction of the particle motion  
   c. no centripetal acceleration for the particle  
   d. none of these **Magnetic Interactions**10. When you consider the interaction of  
   moving charges in terms of the interactions of their magnetic fields, the force on a  
   moving charge is directed a. toward strongest **B** region b. away from strongest **B**  
   region c. parallel to strongest **Bd. parallel to weakest **BElectric and Magnetic  
   Fields**11. Negatively charged particles moving in a region containing perpendicular **B**  
   and **E** fields will be accelerated a. parallel to **Bb. parallel to **Ec. parallel to - **Ed.  
   parallel to - **Be. perpendicular to **B12. If a positive charge at rest is the same distance  
   from a 1 C point charge as it is from a bar magnet producing in 1 Wb/m^2** field, the  
   ratio of the magnetic force to electric force will be a. 1  
   b. **∞**  
   c. undetermined
d. zero
e. $1.6 \times 10^{-9}$

13. Based on their relative strengths, which field would you expect to be the most important in natural processes
   a. gravitational
   b. electric
   c. magnetic

**Magnetic Field Applications**

14. Applications of magnetic fields always involve
   a. permanent magnets
   b. moving charges
   c. monopoles
   d. charges at rest
   e. Hall effect

15. The Hall effect probe on the Viking I spacecraft on Mars enables NASA to measure
   Mar's
   a. atmospheric currents
   b. atmospheric storms
   c. water abundance
   d. magnetic field

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**ALGORITHMIC PROBLEMS**

Listed below are the important equations from this chapter. The problems following the equations will help you learn to translate words into equations and to solve single-concept problems.

**Equations**

$(23.1) F_m = q v \times B$

$(23.2) B = \mu_0 I/l$  \hspace{2cm} \mu_0 = 4\pi \times 10^{-7}$ weber per ampere-meter

$(23.3) B = (\mu_0/4\pi) (i\Delta l \sin \theta/r^2)$

$(23.4) B = \mu_0 I/2r$

$(23.5) B = \mu_0 I/2\pi r$

$(23.6) B = N\mu_0 I/l$

$(23.8) r = mv/Bq$

$(23.11) F = BIl$

$(23.12) \tau = 2F_m d = NBI \sin \theta$

$(23.16) I = k\phi$

$(23.19) V_H = BI/nqd$

**Problems**

1. At the equator the earth's magnetic field is about horizontal and directed from south to north with a strength of $0.5 \times 10^{-4}$ webers/m². Find the force direction and magnitude on a 30-m horizontal wire and carrying a current of 40 A from east to west.

2. If an electron is shot with a speed of $1.00 \times 10^6$ m/sec upward into the earth's magnetic field that has a horizontal component directed north of $1.60 \times 10^{-5}$ Wb/m² (tesla). What is the magnitude and direction of the force acting on the electron?
3. Two long, straight, parallel wires are 4 cm apart and carry a current of 5 A each in the same direction. What is the value of B midway between the two? 4. What is the value of B midway between the wires of problem 3 if the directions of currents are opposite.

5. A solenoid is 1.0 m long; its length is many times greater than its diameter. It has 4000 turns and carries a current of 5.0 A. What is B at the center of the solenoid?

6. It is found that a certain galvanometer has a deflection of 30° for a current of 10 A. What is the value of the torsion constant?

7. The dip angle is 71° at Washington, D.C., and the vertical component of the magnetic field $B$ is $0.543 \times 10^{-4}$ Wb/m$^2$. What is the magnitude of the earth's magnetic induction there?
Answers
1. $6 \times 10^2$ N down 2. $2.56 \times 10^{18}$ N east 3. zero 5. $2.5 \times 10^{-2}$ T 6. $19.0$ A/rad or $00.33$ A/degree 4. $10^4$ Wb/m$^2$ or $10^4$ T 7. $.574 \times 10^{-4}$ T

EXERCISESThese exercises are designed to help you apply the ideas of a section to physical situations. When appropriate the numerical answer is given in brackets at the end of the exercise.

Section 23.3. 1. What quantity is the symbol B used to represent? What are its units in the SI system? Give the SI equivalent of these units in meters, kilograms, seconds, and amperes. 2. Make a sketch of a positive charge traveling at right angles to a uniform magnetic field. Show the force that acts upon the charge and the subsequent path of the charge.

3. Make a sketch of a negative charge traveling at right angles to a uniform magnetic field. Show the force that acts upon the negative charge and the subsequent path of the charge.

Section 23.4. 4. Find the field produced by a current of 10 A at a distance of 5 cm from a very long conductor. [4 x $10^3$Wb/m$^2$] 5. Calculate the field produced at the center of a 200-turn flat coil of 10-cm radius for a current of 5.00 A. [6.3 x $10^3$Wb/m$^2$] 6. What is the magnetic field at the center of an air core solenoid of 1000 turns, 50.0 cm long, and carrying a current of 10.0 A? If this coil is wrapped upon an iron core with permeability of 500, what is the field? [25.1 x $10^3$Wb/m$^2$, 12.6 Wb/m$^2$] 7. Make a sketch of a long, straight, current carrying wire, and show the magnetic field in its vicinity.

8. A solenoid 1.0 m long with 3.0 x $10^3$ turns carries a current of 6.0 A. Calculate the magnetic induction at the center of the solenoid if it has an air core. [.023 Wb/m$^2$]

Section 23.5. 9. An electron of charge $1.6 \times 10^{-19}$ C moves with a speed of $8.0 \times 10^7$ m/sec in a magnetic field of 0.50 (in SI units). a. What is the force on the electron? b. What is the radius of its path? (m_e = 9.1 x $10^{-31}$ kg) c. Make a sketch showing the electron, the field, and its direction of curvature. (Hint: The electron has a negative charge. Assume directions of the motion of the electron and the magnetic field.) [a. $6.4 \times 10^{-12}$ N; b. 9.1 x $10^{-4}$ m]

Section 23.8. 10. A current of 5 A is directed up in a vertical wire in a uniform magnetic field of 1.20 Wb/m$^2$ is directed north. Find the magnitude and direction of the force on a 6-cm section of the vertical wire. [.006 N west] 11. A straight wire 20 cm long and carrying a current of 15 A is placed in a field where the magnetic induction is 0.50 in SI units. a. If the wire and the field are perpendicular to each other, find the force on the wire. b. Sketch the wire, the field, and the direction of the force upon the wire. (Assume whichever directions you wish for the current and the magnetic induction.) [a. 1.5 N] Section 23.9. 12. A rectangular galvanometer coil is suspended in a uniform magnetic field of 0.10 Wb/m$^2$. The coil is 1.0 cm wide and 4.0 cm long, has 200 turns, and carries a current of $1.0 \times 10^8$ A for a deflection of 30°. What is the torsion constant of the suspending system? [7.6 x $10^{11}$ N-m/rad] Section 23.10. 13. A current of 150 A is flowing in a silver ribbon in the x-direction. The thickness of the ribbon is 1.00 mm in the y-direction and 2.00 cm in the z-direction. The magnetic field of 1.20 Wb/m$^2$ is in the y-direction. Assume there are $7.40 \times 10^{28}$ free electrons
per cubic meter. Find:

a. the drift velocity of the electrons
b. the direction and magnitude of the field due to the Hall effect
c. the Hall potential

[a. $6.3 \times 10^{-4}$ m/sec; b. $7.6 \times 10^{-4}$ V/m; c. $1.52 \times 10^{-5}$ V, the + side is in the z-direction]

Section 23.12

14. A mass spectrometer can be used to separate the isotopes of chlorine, $^{35}$Cl and $^{37}$Cl. These ions have masses of $58.45 \times 10^{-27}$ kg and $61.79 \times 10^{-27}$ kg respectively. Assume the chlorine ions enter a common slit after being accelerated through a potential of 10,000 V. At the slit the ions enter perpendicularly a magnetic field of 1.00 Wb/m², and they are turned through an angle of 180° to the detector. What is the separation between the $^{35}$Cl and $^{37}$Cl at the detector? [.48 cm]

**PROBLEMS**

These problems may involve more than one physical concept. When appropriate, the numerical answer is given in brackets at the end of the problem.

15. A velocity selector for accelerated sodium ions can be made by passing the ions through the combination of an electrostatic and a magnetic field. Can you design such an instrument? For what velocity of Na⁺ is your instrument designed? 16. A proton of mass $1.67 \times 10^{-27}$ kg moves with a constant velocity in the earth's magnetic field at the equator of $1.00 \times 10^{-4}$ Wb/m². Find the magnitude and direction of the proton's velocity that will cause the magnetic force to just cancel the weight of the proton $mg$ ($g = 9.80$ m/sec²) [1.02 $\times 10^{-3}$ m/sec east] 17. In a DC motor the magnetic field is shaped such that the long side (12 cm) of the armature winding is always moving perpendicularly to the 0.50 Wb/m² field. What torque will the motor deliver if the 500 turn armature carries a current of 5.0 A and has a radius of 5.0 cm? If the armature revolves at 1800 rpm, what should be the output rating of the motor? [15 N·m, 2800 watts] 18. A long, straight conductor which has a linear density of $5.5 \times 10^{-3}$ kg/m is supported in mid-air by a magnetic field. If the current in the conductor is 15 A, what is the strength of the magnetic field? [3.6 $\times 10^{-3}$ Wb/m²] 19. A 1-m length of wire with a resistance of 12 W is subjected to a voltage of 120 V and experiences an upward force of 2 N. What is the magnitude and direction of the magnetic field in the vicinity of the wire? (Draw a sketch.) [B = 0.2 Wb/m²] 20. Two long, parallel wires are originally 2 cm apart with a current of 20 A in the same direction in each. What is the original force of attraction between these? Plot a curve for the force of attraction as a function of distance between the wires. [4 $\times 10^{-3}$ N/m] 21. A long, iron-cored solenoid which has 5.00 $\times 10^3$ turns per meter carries a current of 5.00 A. The iron has a permeability of 3000.

a. What is the value of the magnetic field at the center of the solenoid? $\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m
b. What happens to the magnetic field if the iron core is removed? What is its final value? [a. $94.2$ Wb/m²; b. $3.14 \times 10^{-2}$ Wb/m²] 22. a. If the electrons in a TV tube are accelerated through 8.20 kV, what is their velocity? $m_e = 9.1 \times 10^{-31}$ kg. b. If these electrons travel 20 cm in a southerly direction in the earth's magnetic field, what happens to them?

c. Calculate the magnitude of the deflection if the vertical component of the earth's magnetic field is 4.55 $\times 10^{-5}$ in SI units. [a. $5.36 \times 10^7$ m/sec; b. deflected to west; c. $3.9 \times 10^{-16}$ N, $d = 29.8 \times 10^{-4}$ m] 23. Make a sketch of the previous problem showing the
motion of electrons in a TV tube. Show the magnetic field direction, the electron direction, the direction of positive current flow, and the directions of any forces on the electrons.

24. At the magnetic equator of the earth the field is north-south with only a horizontal component equal to $5.6 \times 10^5$ in the SI system. It is possible for a current carrying wire of linear density 0.028 kg/m to be prevented from falling. How? What must the direction of the wire be? What current must the wire carry? (Hint: take $g = 9.8$ m/sec$^2$ at the magnetic equator.) [4.9 x 10$^3$ A, E - W E] 

25. A blood flow meter is made to operate on the principle of the Hall effect. The blood flow through a magnetic field of 2 Wb/m$^2$ generates a transverse voltage of 600 microvolts across electrodes separated by 1 mm in the blood stream. Find the velocity of the blood perpendicular to the magnetic field. [0.3 m/sec]

26. Find the force on a stream of singly ionized particles (with speed of $4.0 \times 10^6$ m/sec) that are moving parallel to a wire carrying a current of 10 A. The distance between the ion stream and the center of wire is 2.0 cm. [F = 6.4 x 10$^{-17}$ N]

27. Find the magnitude and direction of the force on an electron moving radially outward from a wire carrying a current I. Assume the velocity of the electron is $v$ at a radial distance $r$ from the center of the wire. [$F = \mu_0 Iev/2\pi r$]

28. A long (length $l$) thin (width $b$) rectangular loop carrying a current $I$ is at a distance $a$ from a long wire (in the plane of the loop) carrying current $2I$. Find the net force on the loop. [$\mu_0 2I^2 l/2\pi (1/a - 1/(a+b))$ toward the wire]