Chapter 9
TRANSPORT PHENOMENA

GOALS
After you have mastered the contents of this chapter you will be able to achieve the following goals:

Transport Equation
Write the quantitative equation for the transport process of a system whose variables are given.

Continuity
State the continuity equation for a system, and explain the flow properties of a system in terms of that equation.

Transport Problems
Use algebraic and graphical methods to solve transport problems for one dimensional systems.

PREREQUISITES
Before beginning this chapter, you should have achieved the goals of Chapter 1, Human Senses, and Chapter 2, Unifying Approaches.
Chapter 9
TRANSPORT PHENOMENA

9.1 Introduction
During heavy rainstorms puddles of water are formed at various places on the ground. The rainwater flows along the ground from one puddle to another and then, perhaps, into a water drainage ditch. What causes the flow of the water along the ground? Have you noticed that water flows more rapidly along some parts of the ground than others? How do you explain the various rates of water flow over various parts of the surface of the earth?

During the serving of a formal dinner in the palace, the silver serving pieces are placed in piping hot food by the servants. The prince discovers that the handle of the silver gravy ladle is rather warm. How do you explain the warmth of the ladle handle? What causes the flow of heat from the gravy to the handle? Meanwhile, the handle of the prince’s china coffee cup remains cool even though his cup contains hot coffee. How do you explain the different rates of heat flow?

Can you take a dry-cell battery, two pieces of wire, and a flashlight bulb and attach them together so that the bulb will light? How do you do it? How do you explain what has happened? Do you use the flow of something to explain this phenomena? If so, what do you say is flowing? What causes the flow to occur?

These are just three examples of a more general phenomena that we observe. The universe in which we live is always changing. Something moves from one part of the universe to another part. These movements are the primary concern of this chapter.

9.2 Temperature Differences and Gradients

In these three examples each of the systems had at least one variable that had different values in various parts of the system. Take the case of the prince and the silver serving pieces of the palace. The temperature varied from one part of the palace to another. In particular the hot food is at a much higher temperature than its surroundings. The silver ladle that was placed in the gravy had different temperatures at its two ends see Figure 9.1. We can compute the rate at which the temperature changes with distance along the ladle handle by taking the difference in the temperatures between the ends of the ladle and dividing that by the length of the handle to give the rate of change of temperature with distance = \((T_2 - T_1)/L\) where \(T_2\) is the temperature of the gravy, \(T_1\) is the temperature of the cool end of the ladle handle, and \(L\) is the length of the handle.

This represents an average value for the rate of temperature change over the length of the handle. If the gravy is at a temperature of 82°C and the cool end is at 33°C and the
Physics Including Human Applications

Handle is 7 cm long, the rate of change of temperature with distance = (82 - 33)°/7 cm = 7°C/cm. The temperature of the handle decreases 7° for every centimeter you move away from the gravy. There is no obvious reason why the temperature should change at a constant rate all along the ladle handle. In fact, shortly after the ladle is dipped into the hot gravy the temperature as measured along the ladle handle might be represented by the curve in Figure 9.2.

In this case the rate of change of temperature with distance varies along the handle. As you will recall from your experiences with graphs, the rate of change of the y variable with respect to x is the definition of the slope of the curve. The slope is found by taking the ratio of the differences between the ordinates and the abscissa of two points on the straight line tangent to the curve at the point of interest. In Figure 9.2 the value of the slope at the point 3 cm from the hot end is computed to be (84 - 48)/(1 - 5) = 36/4 = 9°C/cm. You can see from this figure that the rate of change of the temperature with distance does not need to be a constant for all values of the distance, but in fact the rate of change of the temperature with distance may have a different value at each point along the handle of the ladle. The rate of change of temperature with distance is called the temperature gradient. The magnitude of the gradient, as you may recall from Chapter 2, is defined as the rate of change of a variable of a system with respect to distance. Since a variable of a system may change differently in the x and y directions, it is necessary to specify the direction of the gradient of a variable. We will use the following notation where grad_x T is a symbol for the gradient of the temperature with respect to changes in the distance in the x direction, ΔT is the symbol for the change in temperature(T_2 - T_1), and Δx is the symbol for the change in distance (x_2 - x_1). As you know from your experience in calculating the slopes of curves on graph paper, if the change in x is made sufficiently small, the value of ΔT/Δx will be equal to the slope of the line tangent to the T versus x curve at the point of interest. In Figure 9.2 the grad_x T at 3 cm is given by the slope of the line tangent to the curve at that point, hence the grad_x T at 3 cm is equal to -9°C/cm.

In summary, the magnitude of the gradient of a variable of the system is the ratio of the change in value of that variable to change in distance. If a variable of a system has the units of joules, what will be the units of the gradient of that variable? If a variable has a definite value at each point in the system, then the gradient of that variable will also have a definite value at each point in the system. For most real systems both the values of the variables and of the gradients are smoothly changing quantities over the volume of a system.
EXAMPLE

Let us take the contour map of the hill shown in Figure 9.3 and estimate the components of the gradient of the elevation in given directions for various positions on the hill. Where is the $\text{grad}_E H$ (gradient of height in the easterly direction) the greatest? The least? Zero. What is the relationship between the $\text{grad}_E H$ and $\text{grad}_W H$ (gradient of the height in the westerly direction)? Where is $\text{grad}_S H$ (gradient of the height in the southerly direction) greatest? The least? Zero? Where do you predict that water flowing down the hill will have the greatest rate of flow?

![Figure 9.3](image)

So far we have only considered the temperature of the ladle handle at one particular time. What happens to the temperature of the ladle handle as time goes on?

Questions

1. What does a negative value for the rate of change of temperature with distance indicate?

2. In Figure 9.2 is the rate of change of temperature with distance greater or less at 4 cm than at 3 cm? At 5 cm than at 3 cm?

3. How do you answer these questions quickly by looking at the graph and without making a quantitative calculation?

4. You can imagine that Figure 9.2 represents a one-dimensional contour map of the temperature of the ladle handle. Where does the $\text{grad}_x T$ have its largest value? Smallest? Is it zero anywhere along the ladle handle?

9.3 Flows and Currents

It is natural that when we encounter the continuous motion of something from one part of the universe to another, we should evoke the language and images of those continuously moving objects with which we are most familiar.

So you will find that the subject of transport phenomena is couched in terms that are used in everyday life to describe the motion of water. The actual transport phenomena that we are describing may, in fact, bear little resemblance to the flow of water, yet we will be using terms that we first knew as we poured water from one jar to another in a bathtub, or as we stood beside a puddle of water that fed the trickle of water draining down our driveway. In those situations we remained in one location and watched the quantity of water move past us in time. This quantity of water flow may be called water current (quantity of water per second).
9.4 Heat Flow

If we now apply this imagery to the hot gravy system in the palace, we may imagine that the hot gravy contains "heat" which flows along the ladle handle. Then we can define the heat flow as the rate of change of the quantity of heat with respect to time. We will use the symbol $I$ for quantity of flow, and we can write the following equation for heat flow:

$$I \text{(heat)} = \frac{\text{change in quantity of heat}}{\text{change in time}} = \frac{\Delta H}{\Delta t}$$

(9.1)

As a specific example, suppose we have a way of measuring the amount of heat that is contained in a small portion of the ladle handle that is located 3 cm from the gravy. Then the quantity of heat contained in that small portion of ladle handle might vary with time according to the graph in Figure 9.4.

We can calculate the heat flow from the slope of the line tangent to the current at the time we chose. For example, let us select the time as $t = 4$ sec, then from the graph we can compute $I$

$$I = \frac{(5.9 - 1.5)}{(5.0 - 3.0)} \text{ mJ}/\text{sec} = 2.2 \text{ mJ/sec}$$

(9.2)

where $I$ is the heat flow at a distance of 3 cm from the gravy at the time of 4 sec after the ladle is put into the gravy. This phenomenon may be represented pictorially as in Figure 9.5, where $x$ is the distance along the handle starting from the surface of the gravy and $H$ is the amount of heat contained in the portion of the handle bounded by the solid lines in the figure. The heat flow $I$ in Figure 9.5 is shown as if the change in the quantity of heat in that portion of the handle is completely explained by the flow of heat along the handle."
Questions

5. What are the ways other than along the handle that the heat might escape from that portion of the handle?

6. Does it seem reasonable from your own experience with metal rods held in a fire that the heat flow along the rod is much, much greater than the flow of heat in other directions?

9.5 Water Flow

Let us consider another familiar example of flow, filling a bucket of water from a garden hose. Suppose you have a 19-liter (5-gallon) bucket that you wish to fill with water. The graph of the amount of water in the bucket at any time after you have turned on the hose might be like the one given in Figure 9.6.

**FIGURE 9.6**
Graph of the quantity of water in a bucket vs. time of flow.

In order to use this graph to calculate the flow of water through the hose, what assumptions must you make about water? About the hose? What is the water flow at a time equal to 200 sec? We can answer this question using the slope of the line tangent to the curve in Figure 9.6 at \( t = 200 \) seconds:

\[
I \text{(water)} = \frac{\text{change in amount of water}}{\text{change in time}} = \frac{\Delta Q}{\Delta t} \tag{9.3}
\]

At \( t = 200 \),

\[
I = \frac{18.0 - 8.0 \text{ liters}}{250 - 150 \text{ sec}} = 0.10 \text{ liters/sec}
\]

Questions

7. How can you describe the water flow between 150 and 200 sec?

8. What two explanations can you give for the curve for times greater than 300 sec?

9.6 Current Density and Continuity

Is it possible to have a water flow if the amount of water in the bucket does not change with time? If you said yes, you are right. Consider the case of a bucket with a hole in the bottom that allows water to leak out of the bottom at exactly the same rate we are
adding water in the top. The total amount of water in the bucket remains the same, but there is a flow of water through the bucket. So we see that our previous definition, while straightforward if the change in amount of time is not zero, does lead us astray in some cases. Let us, therefore, proceed to clear up this messy business with a more precise and more formal definition of a property of the system that we can call the current density and represent by the symbol $J$. Let us take any system and construct an imaginary plane of area $A$. Then we shall measure the quantity $Q$ of something that passes through that plane in a given time $t$, and we will define the magnitude of the current density by the ratio

$$J = \frac{Q}{At} \quad (9.4)$$

Hence, $J$ will have the units of the quantity of something divided by the product of area times time, in SI units quantity divided by $m^2 \cdot \text{sec}$. It is possible that $J$ will depend upon the location of our imaginary plane. So we need to define a direction for $J$ which we will take to be perpendicular to the plane $A$ as shown in Figure 9.7. Then we can indicate the current density in the $x$-direction by $J_x$, which will represent the quantity of something that passes through an area $A$ of the $yz$ plane in a time $t$.

To examine the properties of current density in detail, let us consider a bucket (see Figure 9.8) which has an input tube of area $A_1 (m^2)$ and an outflow tube of area $A_2$. Say the bucket has an amount of water $Q_0 (\text{kg})$ in it at time zero and that there is an inflow current density of water of $J_1$ and an outflow current density of $J_2 (\text{kg} / m^2 \cdot \text{sec})$. From the definition of current density we can show that the amount of water flowing into the bucket in $t$ seconds is given by the product of the input current density times the area of the input tube times the time.

$$\text{inflow} = J_1 A_1 t \quad (9.5)$$

The amount flowing out of the bucket in $t$ seconds is given by the product of the outflow current density times the area of the outflow tube times the time.

$$\text{outflow} = J_2 A_2 t \quad (9.6)$$
The net increase in the amount of water in the bucket after \( t \) seconds will be the difference between inflow and outflow:

\[
\text{net increase} = J_1 A_1 t - J_2 A_2 t = (J_1 A_1 - J_2 A_2) t
\]

(9.7)

Since at the start there was a quantity \( Q_0 \) of water in the bucket, then the total amount of water \( Q \) in the bucket at time \( t \) is given by the sum of \( Q_0 \) and the net increase:

\[
Q = Q_0 + (J_1 A_1 - J_2 A_2) t
\]

(9.8)

The net change in the amount of water in the bucket in a time of \( t \) seconds is given by

\[
\Delta Q / \Delta t = Q - Q_0 / t = J_1 A_1 - J_2 A_2
\]

(9.9)

We see clearly in this equation that if the net flow is zero, that is, \( \Delta Q / \Delta t = 0 \), it is not the total current that must be zero, but rather \( J_1 A_1 \) must be equal to \( J_2 A_2 \). That is, the amount of water flowing into the bucket must be equal to the amount of water flowing out of the bucket. The amount of water in the bucket remains constant. This is another example of a system in equilibrium as introduced in Chapter 2.

We can use this same intuitive concept to write down a statement in mathematical form for the conservation of matter, which we will call the continuity equation. Let us consider a very small imaginary cubical volume of a liquid (Figure 9.9). We have seen above that the time rate of change of the amount of matter in that small volume is given by the difference between the inflow and the outflow,

\[
\Delta Q / \Delta t = (J_1 - J_2) a
\]

(9.10)

where \( a \) is the area of one face of our small cube. The difference between the inflow and the outflow is the change in the current density \( \Delta J \). We then rewrite Equation 9.10 as follows,

\[
Q / t = (\Delta J)a
\]

(9.11)

In order to obtain an expression that is independent of the volume of our imaginary cube, we will divide this equation by the volume of the cube \( V \). The result will be an expression of the change in quantity in a unit volume in a second,

\[
(1/V) \Delta Q / \Delta t = 1/V (\Delta J)a
\]

(9.12)

The quantity (mass) of a liquid found in a unit volume is called the density of the liquid, \( Q / V = r \). The volume of the imaginary cube is equal to the area \( a \) times the length of a side of the cube \( l \), \( V = al \). We can use these facts to change the form of Equation 9.12,

\[
(1/V) \Delta Q / \Delta t = \Delta \rho / \Delta t = (1/V) (\Delta J)a = \Delta J / l
\]

\[
\Delta \rho / \Delta t = \Delta J / l
\]

(9.13)

where \( \Delta \rho \) is the change of the density of the liquid. This equation is called the continuity equation.

Hence, the continuity of matter can be stated as follows:

*The time rate of change of the amount of material in a unit volume of the material is equal to the change of the current density with respect to distance.*

In other words, if the amount of the material in a unit volume is changing there must be some net flow of material. We can restate this in a manner very similar to the statement of the conservation laws: If the density of matter in a given volume is not changing, we know that the current density is constant, i.e., \( J \) does not change; so the inflow current in
the volume is equal to the outflow current from the volume. For incompressible liquids
the change of liquid density with time is assumed to be zero, \( \Delta \rho / \Delta t = 0 \); so the flow of
matter is continuous, \( \Delta J = 0 \).

9.7 How to Increase the Flow

What is it that gives rise to the various rates of flow? We already have a vague idea
about that. We know that the cool end of a metal rod placed in a fire gets hot faster than
the cool end of a metal rod placed in hot water. We have experienced the fact that heat
seems to flow away from very hot objects faster than it does from less hot objects. There
seems to be some relationship between the rate of heat flow along the rod and the
difference in temperature between the two ends of the rod. The greater the temperature
difference, the faster heat will flow. Let us, therefore, make the simplest possible
assumption, that is, that the heat current density is directly proportional to the
temperature gradient.

\[
J_{(heat)} \propto \frac{\Delta T}{\Delta x} \tag{9.14}
\]

Of course, you may say, how can such a simple assumption be justified? You are right,
but the test of any assumption is the test of experiment. The question is, does this
simple model, with a current density directly proportional to the gradient of a system
variable, help in explaining transport phenomena? Does it give us a feeling of
understanding current? Does it allow us to make reasonably accurate predictions about
the current in systems that we have not yet tested?

We can change the above relationship to an equation by defining a proportionality
constant to relate the current to the temperature gradient. In Figure 9.10 you can see
that the heat flow is in the positive \( x \) direction if the high temperature end of the rod is
located at \( x = 0 \). Therefore, the change in temperature with distance will be negative
when the heat flow is positive, and we will have a negative proportionality constant in
our equation:

\[
J_{(heat)} = -K_H \frac{\Delta T}{\Delta x} \tag{9.15}
\]

where \( K_H \) is positive and is called the \textit{coefficient of thermal conductivity}.

So far we have discussed primarily heat and water flow. We have been able to
develop some concepts that seem to be useful in understanding the flow of heat from
one place to another. We have defined the gradient of a variable as a quantity with both
size and direction. The size of the gradient of a variable is given by the ratio of the
change in the variable to the change in distance. We have defined a current density as
the amount of something that passes through a plane of unit area in 1 sec. We have
found that we can write a continuity equation which shows that the time rate of change
in the density of matter in some region of space must be equal to the difference between
the inflow and outflow of matter in that region. Finally, we have proposed that the
current density is directly proportional to the proper gradient, and that the
proportionality constant that relates the current to the gradient is negative. Now let us
use these concepts to examine some other transport systems.
Questions

9. If the quantity of heat is measured in joules, what are the units for the current
density for heat flow in SI units? __________

10. If the temperature is measured in degrees celsius, what are the units for the
temperature gradient in SI units? __________

11. What are the units of coefficient of thermal conductivity?

12. If the coefficient of thermal conductivity for a silver gravy ladle is 420 in SI units,
what current densities of heat flow can you calculate from Figure 9.2 at distances
of 1 cm, 3 cm, and 5 cm from the gravy?

9.8 Diffusion

How does oxygen get into the single cells within the human body where it is essential
for life processes? Outside the cell there exists a concentration of oxygen $c_0$ and inside
the cell there is a different and lower concentration of oxygen $c_c$ (see Figure 9.11).
Hence, there is a gradient in oxygen concentration across the boundary of the single
cell. We might expect some flow of oxygen across the boundary. We are now quite
prepared to deal with the so-called Fick’s law for oxygen diffusion, which is stated as

\[ J = -D \left( \frac{\Delta c}{\Delta x} \right) \]  

(9.16)

where $J$ is the current of oxygen molecules in moles/m$^2$ sec and $c$ is the concentration of
oxygen molecules in moles/m$^3$ and $x$ is the distance in meters. The proportionality
constant $D$ is called the diffusion constant.

This equation is simply another form of the relationship we previously developed to
relate a current to a gradient. To solve for the rate of diffusion of oxygen into a cell, let
us consider a plane surface for the cell boundary and take the diffusion constant for the
cell to be $D_c$ and the diffusion constant for the material exterior to the cell to be $D_0$. Then
the current of oxygen is given by (see Figure 9.11),

\[ J = -(D_0 / \Delta x) \left( c_i - c_0 \right) \]  

(9.17)
in the exterior material and by

\[ J = -(D_c / \Delta x) \left( c_c - c_i \right) \]  

(9.18)
in the cell where $c_i$ is the concentration of oxygen molecules at the cell wall. To solve for
the diffusion current of oxygen molecules we simply eliminate the $c_i$ term from these
two equations as follows: Solve Equation 9.17 for $c_i = -J \Delta x / D_0 + c_0$ substitute that for $c_i$
in Equation 9.18, and solve for $J$. This results in the following expression:
\[ J = \frac{D_0 D_c (c - c_0)}{(D_0 + D_c) \Delta x} \]  
(9.19)

If we know the interior and exterior concentration of oxygen molecules, we can use Equation 9.19 to compute the rate of flow of oxygen molecules through the surface of a cell. We can also combine Equation 9.19 with the continuity equation to find the concentration of oxygen molecules in the cell as the time changes. The continuity equation, which assumes that no oxygen molecules are being generated or used inside the cell, may be written from Equation 9.13 as,

\[ \frac{\Delta c}{\Delta t} = \frac{J}{\Delta x} \]

or
\[ J = \Delta x \frac{\Delta c}{\Delta t} \]  
(9.20)

This statement combined with Equation 9.19 enables us to show that the time rate of change of the concentration of oxygen in the cell is directly proportional to the concentration of oxygen in the cell,

\[ \frac{\Delta c}{\Delta t} \propto c - c_0 \]  
(9.21)

From the section on exponential functions in the mathematics supplement of this book, Equation 9.21 has a mathematical solution given by the following equation,

\[ c - c_0 = (c_k - c_0) e^{-at} \]  
(9.22)

where \( a = \frac{D_0 D_c}{(D_0 + D_c) \Delta x^2} \), \( c_k \) is the initial concentration of oxygen inside of the cell, \( \Delta x \) is a characteristic size of the cell and \( e \) is the base number for natural logarithms. This result is shown graphically in Figure 9.12.

**Questions**

13. What are the units of \( D \) in the SI units?

14. Check the dimensional consistency of Equation 9.19. Does the current \( J \) still have the same units as in the defining Equation 9.16?
SUMMARY

Use these questions to evaluate how well you have achieved the goals of this chapter. The answers to these questions are given at the end of this summary with the number of the section where you can find the related content material.

Transport Equation

1. The quantity of matter transported per unit of time by diffusion is defined as \( q = \frac{\Delta q}{\Delta t} \):
   - a. \( \frac{\Delta q}{\Delta t} \)
   - b. \( \frac{\Delta q}{\Delta x} \)
   - c. \( \Delta q \)
   - d. \( \Delta t \)
   - e. none of these

2. The matter gradient is defined as
   - a. \( \frac{\Delta q}{\Delta t} \)
   - b. \( \frac{\Delta q}{\Delta x} \)
   - c. \( \Delta q \)
   - d. \( \Delta t \)
   - e. none of these

3. In diffusion the flow of matter is directly proportional to
   - a. \( \frac{\Delta q}{\Delta x} \)
   - b. \( -\frac{\Delta q}{\Delta x} \)
   - c. \( \frac{\Delta q}{\Delta t} \)
   - d. \( -\frac{\Delta q}{\Delta t} \)
   - e. none of these

Continuity

4. The continuity equation can be derived from the unifying approach involving
   - a. superposition
   - b. conservation
   - c. inertia
   - d. fields
   - e. resonance

5. The continuity equation predicts that if more of a quantity flows out of a volume than flows into the volume then
   - a. physics is violated
   - b. there is a source inside the volume
   - c. there is a sink inside the volume
   - d. the volume is a sphere
   - e. volume numerically equals area
Transport Problems

6. If the temperature gradient across a window pane is doubled while other variables remain unchanged, the heat current density through the window will
   a. quadruple
   b. be cut in half
   c. double
   d. remain the same
   e. none of these

7. Which of the following processes have quantity transport proportional to the surface area of the system?
   a. thermal conduction
   b. water flow
   c. diffusion
   d. all of these
   e. none of these

8. Which of the following ratios is constant for a given window pane?
   a. heat current density / temperature gradient
   b. temperature change / window thickness
   c. change in temperature / change in time
   d. all of these
   e. none of these

Answers

1. a (Section 9.8)
2. b (Section 9.8)
3. b (Section 9.8)
4. b (Section 9.6)
5. b (Section 9.6)
6. c (Section 9.7)
7. d (Section 9.8)
8. a (Section 9.7)

ALGORITHMIC PROBLEMS

Listed below are the important equations from this chapter. The problems following the equations will help you learn to translate words into equations and solve single concept problems.
Equations

\[ I \text{(heat)} = \frac{\Delta H}{\Delta t} \quad \text{(9.1)} \]

\[ I \text{(water)} = \frac{\Delta Q}{\Delta t} \quad \text{(9.3)} \]

\[ J = \frac{Q}{At} \quad \text{(9.4)} \]

\[ \frac{\Delta \rho}{\Delta t} = \frac{\Delta J}{l} \quad \text{(9.13)} \]

\[ J_{\text{(heat)}} = -K_{tt} \frac{\Delta T}{\Delta x} \quad \text{(9.15)} \]

\[ J = -D \frac{\Delta c}{\Delta x} \quad \text{(9.16)} \]

Problems

1. If \(1.0 \times 10^6\) J of heat are conducted through a window pane per hour, what is the rate of flow of heat?

2. If 12 liters of water flow into a bucket in 30 sec, what is the rate of flow?

3. If the diffusion constant for certain molecules in water is \(1.0 \times 10^{-9}\) m\(^2\)/sec and concentration gradient is \(5.0 \times 10^4\) kg/m\(^3\) per m across a membrane. Find the diffusion current density across the membrane.

4. The window in a room has a thickness of glass of 2.0 mm. The room temperature is 20.0°C and outside temperature is 0.0°C. The thermal conductivity of glass is 1.0 J/(sec-m-deg). Calculate the rate of heat loss per unit area through the window.

5. During the heating of the air in a hot-air balloon, the density of air inside the balloon changes from 1.29 kg/m\(^3\) to 1.09 kg/m\(^3\) in 10 min. What is the average net current of air from the balloon per unit volume?

Answers

1. \(10^6\) J/hr

2. 24 liters/min

3. \(5.0 \times 10^{-5}\) kg/m\(^2\)-sec

4. \(1.00 \times 10^4\) J/sec m\(^2\)

5. 0.02 kg/m\(^3\) min
EXERCISES

These exercises are designed to help you apply the ideas of a section to physical situations. Where appropriate the quantitative answer is given at the end of each exercise.

Section 9.2

1. A contour map for Mt. Taum Sauk, the highest mountain in the state of Missouri is shown in Figure 9.13. Compute the height gradients from this map from west to east and from south to north. Sketch how the mountain looks from the West and from the South.

Section 9.6

2. During an iron ore smelting process the quantity of material in a large crucible is continuously monitored. The process engineer notes that the mass of material in the crucible is 4.2 x 10^3 kg at 10 A.M. and 2.8 x 10^3 kg at 10:20 A.M. The volume of crucible is 70 m^3. (a) What has happened during the time from 10 A.M. to 10:20 A.M.? (b) What is the change in density of all matter in the crucible during this time? (c) What is the average current during this time? [a. 1.4 x 10^2 kg effused; b. decreases 20 kg/m^3; c. 70 kg/min outward.]

3. A service station operator was checking the antifreeze in the radiator of your automobile with a hydrometer. She noticed that the specific gravity of the solution was only 1.05. She added 4 liters of antifreeze of specific gravity 1.80, which just filled the 20-liter radiator. She noticed that the final specific gravity of material in the radiator was 1.30, and she exclaimed, "Your car has a leak in the radiator." Assume the station operator used the continuity equation to arrive at her conclusion. Explain her reasoning. [If no loss, the final specific gravity should have been 1.20.]

Section 9.7

4. An iron rod is placed into a forger's fire. The hot end reaches a temperature of 1000°C. If the rod is 100 cm long and the room temperature is 30°C, what is the temperature gradient along the rod? The thermal conductivity of iron is about 4 J/sec-cm-°C. The diameter of the rod is 2.5 cm. What is the heat flow along the rod? [9.7°C/cm, 190 J/sec]

5. The handle of a silver spoon 5 mm in diameter conducts heat along its handle at the rate of 5 mJ/sec. The thermal conductivity of silver is 4.2 mJ/sec-cm-°C. What is the temperature gradient along the handle? [6.1 °C/cm]
6. The distribution of oxygen molecules in the bloodstream of a patient is given by the expression \( c = c_o + ax^2 \) where \( x \) is the distance from the heart, \( c_o = 3 \times 10^{19} \) molecules/cm\(^3\), and \( a \) is a constant with a value of \( 3 \times 10^{15} \) molecules/cm\(^5\).

a. Compute the concentration of oxygen molecules 10 cm, 50 cm, and 100 cm from the heart.

b. Compute the gradient of the oxygen concentration at 10 cm, 50 cm, and 100 cm from the heart.

c. The diffusion constant for oxygen in the blood is \( 2 \times 10^{-5} \) cm\(^2\)/sec. What is the oxygen current flow at 10 cm, 50 cm, and 100 cm from the heart?

\[ a. \text{ at } 10 \text{ cm } c = 3.03 \times 10^{19} \text{ molecules/cm}^3; \text{ at } 50 \text{ cm } c = 3.75 \times 10^{19} \text{ molecules/cm}^3, \text{ at } 100 \text{ cm } c = 6 \times 10^{19} \text{ molecules/cm}^3; \]
\[ \text{b. at } 10 \text{ cm } \Delta c/\Delta x = 6 \times 10^{16} \text{ molecules/cm}^4, \text{ at } 50 \text{ cm } \Delta c/\Delta x = 3.0 \times 10^{17} \text{ molecules/cm}^4, \text{ at } 100 \text{ cm } \Delta c/\Delta x = 6 \times 10^{17} \text{ molecules/cm}^4; \]
\[ \text{c. at } 10 \text{ cm } J_x = 1.2 \times 10^{12} \text{ molecule/sec, at } 50 \text{ cm } J_x = 6 \times 10^{12} \text{ molecule/sec, at } 100 \text{ cm } J_x = 1.2 \times 10^{13} \text{ molecule/sec} \]

**PROBLEMS**

These problems may involve more than one physical concept.

7. The ratio of the diffusion coefficient of KCl to that of NaCl is 1.46. The concentration difference for KCl across a membrane is one-half the equilibrium value. Find the concentration difference for NaCl that will produce the same diffusion rate. [73 percent of the equilibrium concentration.]

8. Show that Equation 9.15 becomes

\[ J_h = -\frac{(T_b - T_a)}{(x/K_1 + x/K_2)} \]

for two layers each of thickness \( x \); \( K_1 \) is thermal conductivity of one layer and \( K_2 \) is thermal conductivity for the other layer. There is a general equation for any number of layers \( N \) as follows:

\[ J_h = -\frac{(T_b - T_a)}{\sum_{i=1}^{N} (\Delta x/K_i)} \]

Use this equation to find the amount by which heat loss can be reduced by using good storm windows that provide a dead air space of thickness \( 5d \) where \( d \) is the glass thickness and \( K_{\text{air}} = (1/40) K_{\text{glass}} \).

9. A styrofoam ice chest is 20 x 40 x 50 cm and 5 cm thick. The chest contains 5 kg of ice (at 0°C). The chest is located in a hot room of 31°C. How long will it take for 1 kg of the ice to melt if the thermal conductivity of styrofoam is \( 1.6 \times 10^4 \) watts/deg-cm? (Hint: 1 g of ice absorbs 336 J of heat energy upon melting.) [The inside area of the box is 4400 cm\(^2\). The thermal gradient in the styrofoam is 6.2°C/cm. The heat current is \( 9.9 \times 10^3 \) watts m\(^2\) or 4.36 watts into the ice chest. So 1 g of ice melts in 1.28 minutes. Then 1 kg of ice melts in 21.4 hours.]
10. A circular pond is 1 m deep and 100 m in radius. Suppose we introduce a bottom feeding fish. The fish respires $M$ g of oxygen per second. Find the necessary diffusion coefficient for oxygen in water if the fish is to survive on oxygen diffusing from the surface where the concentration is $8M$ g of oxygen per cubic meter of water, and the concentration of oxygen at the bottom is $1M$ g per cubic meter. $[4.55 \times 10^{-6} \text{ m}^2/\text{sec}]$

11. Two compartments in a tank are separated by an artificial membrane of 1 mm thickness. In compartment A is a solution with 10 g of glucose per liter, and in compartment B is a solution with 6 g of glucose per liter. The total area of the membrane is 2.0 cm$^2$, and the pores constitute 12.5 percent of the total area. The molecular weight of glucose is 180. If the diffusion coefficient $D = 3 \times 10^{-6}$ cm$^2$/sec, what is the current density? What is the original rate of flow through the membrane? $[J = 0.67 \times 10^9 \text{ molecules/cm}^2\text{-sec}; \text{rate} = 1.34 \times 10^9 \text{ molecules/sec}]$

12. During a blood plasma (specific gravity 1.21) transfer of 1.00 liter, a doctor noticed that the specific gravity of the patient's blood increased from 1.03 to 1.08. (a) What was the initial quantity of blood in the patient? (b) How could a doctor use a plasma transfer to detect internal bleeding in a patient? [3 liters]