Chapter 22
Basic Electrical Measurements

GOALS
When you have mastered the contents of this chapter, you will be able to achieve the following goals:

Definitions
Define each of the following terms, and use it in an operational definition:
- ampere
- electrical conductivity
- ohm
- resistivity
- electromotive force (emf)
- Seebeck effect
- Peltier effect
- piezoelectric effect

Resistors
Determine an equivalent resistance for a series or parallel combination of resistors.

Ohm's Law
Solve problems using the relationship among resistance, potential difference, and electric current—that is, apply Ohm's law to simple circuits.

Power Loss
Solve problems for instantaneous power in resistive elements obeying Ohm's law.

DC Circuits
Analyze direct-current circuits consisting of resistances and sources of emfs, and find the currents, terminal potential difference of sources of emfs, potential drops, and power developed in circuit elements.

DC Instruments
Explain the basic principle of operation of direct-current instruments: potentiometer, Wheatstone bridge, and thermocouple.

Bioelectricity
Describe some application of electricity in human physiology.

PREREQUISITES
Before you begin this chapter, you need to have mastered the basic concepts of Chapter 5, Energy, Chapter 11, Thermal Transport, and Chapter 21, Electrical Properties of Matter.
# Chapter 22
## Basic Electrical Measurements

**OVERVIEW** - Most of the everyday applications of electricity involve the flow of electric charges through circuits. In this chapter, many of the basic properties of electrical circuits will be discussed. From a simple discussion of Ohm’s and Joule's Law in section 22.4, more sophisticated circuits are introduced. The remaining parts of the chapter deal with either electrical measuring devices or electrical physiology.

**SUGGESTED STUDY PROCEDURE** - To begin your study of this chapter, read the following Chapter Goals: Definitions, Resistors, Ohm's Law, Power Loss, DC Circuits, and Bioelectricity. Each term listed under the Goal of Definitions is discussed in the first section of this Study Guide chapter.

Next, you should carefully read through the following chapter sections and work through the examples provided at the end of each: 22.1-22.4, 22.8-22.11, and 22.13. Questions you encounter during your reading are answered in the second section of this Study Guide chapter.

At the end of the chapter, read the Chapter Summary and do Summary Exercises 1-11, 14 and 16. Check your answers carefully and compare them with those given at the end of the section. Next, do Algorithmic Problems 2-7 and Exercises and Problems 3, 7, 13, 17, and 18. Now, for more practice with the concepts of DC Circuits, complete each of the problems presented in the Examples section of this Study Guide chapter.

After completing the above procedure, you should be prepared to attempt the Practice Test on Basic Electrical Measurements found at the end of this Study Guide chapter. After you have completed each part, check your answers against those given at the end of the test. If you were unsuccessful in any of the areas, refer back to the text or to this chapter for additional assistance. This study procedure is outlined below.

<table>
<thead>
<tr>
<th>Chapter Goals</th>
<th>Suggested Text Readings</th>
<th>Summary Exercises</th>
<th>Algorithmic Problems</th>
<th>Exercises &amp; Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definitions</td>
<td>22.1, 22.2, 22.8</td>
<td>1-8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>22.9, 22.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistors</td>
<td>22.3</td>
<td>9</td>
<td>2, 3, 4</td>
<td></td>
</tr>
<tr>
<td>Ohm's Law</td>
<td>22.4</td>
<td>10</td>
<td>5</td>
<td>3, 7</td>
</tr>
<tr>
<td>Power Loss</td>
<td>22.4</td>
<td>11</td>
<td>7</td>
<td>17, 18</td>
</tr>
<tr>
<td>DC Circuits</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bioelectricity</td>
<td>22.11, 22.13</td>
<td>14, 16</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>DC Circuits</td>
<td>22.14</td>
<td>12</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>DC Instruments</td>
<td>22.5, 22.6, 22.7</td>
<td>13</td>
<td></td>
<td>8, 9, 10, 12</td>
</tr>
</tbody>
</table>
DEFINITIONS

**AMPERE** - A measurement of the flow of charges through an electrical conductor. The passage of one coulomb of electric charge in one second is one ampere.

**ELECTRICAL CONDUCTIVITY** - The proportionality constant between current density and potential gradient.

**OHM** (unit of resistance) - The OHM is the resistance that has a potential difference of one volt across it when it carries a current of one ampere. Each individual component of an electrical circuit will display a unique resistance to the flow of electrical charges.

**RESISTIVITY** - The reciprocal of conductivity with units of ohms-meter. This constant reflects a general resistance property for a material like copper or lead. It is not dependent upon the length and/or cross-sectional arrangement of a circuit component.

**ELECTROMOTIVE FORCE** (emf) - The potential difference across a source of electrical energy with no current flow. An emf of a battery is a voltage measurement when the current flow through the battery is zero. Thus the terminal voltage has a maximum value.

**SEEBECK EFFECT** - The physical process that produces the emf generated by junctions of dissimilar metals when they are heated or cooled, (e.g., thermocouples).

**PELTIER EFFECT** - The process characterized by the generation or absorption of heat at a junction of dissimilar metals when a current passes through the junction. (The reverse of the Seebeck effect.) An electric current flowing through the junction of dissimilar metals produces either a liberation or absorption of heat (depending upon the direction of the current) in direct proportion to the quantity of charge passing through the junction.

**PIEZO ELECTRICITY** - The ability of certain crystals (e.g., quartz) to generate an emf when subjected to mechanical strain. A common crystal microphone operates as a mechanical energy-electrical energy transducer using this effect.
ANSWERS TO QUESTIONS FOUND IN THE TEXT

SECTION 22.1 Introduction
The proliferation of electrical appliances has become a hallmark of the United States. Many electrical energy supply systems have a procedure for charging less per unit of energy the more energy you use. The economics of the system has encouraged the development of a vast array of electrical devices from electric toothbrushes to electric garage door openers.

A typical person in a single family house may use about 10 units of electrical energy per day. The units used to measure electrical energy by the electrical energy supply systems are kilowatt hours rather than joules. There are 3.6 x 10^6 joules in one kilowatt hour.

SECTION 22.2 Electrical Charges in Motion
The resistivity $\rho$ (rho) is typically measured in the units of ohm $\cdot$ centimeters. Questions - 1 and 2. Since the electrical charge of an electron is $1.6 \times 10^{-19}$ C, if $6.25 \times 10^{18}$ electrons passed a location in one second, there would be an electrical current of one ampere passing that location. If the current density were 1A/m$^2$, then $6.25 \times 10^{18}$ electrons would be passing through an area of 1 square meter or $10^4$ square centimeters. The current density of 1A/m$^2$ is equal to $6.25 \times 10^{14}$ electrons per square centimeter.

3. If the potential difference is linear with distance along the circuit then the total potential difference from end to end divided by the total length will be equal to the ratio of the change in voltage to change in displacement for any portion of the circuit. If we rewrite Equation 22.3 in terms of the electric field it becomes $J = sE$.

4. A good conductor will have a small value of resistivity.

SECTION 22.3 Sources of Electrical Energy
If you measure the internal resistance of a battery then you find low resistance means a good battery and high resistance of the same kind of battery means a bad, or dead, battery.

5. If various portions of electrostatic systems are connected by conducting materials, then electric charge can always flow in response to electric potentials and no large electrostatic potentials can develop.

SECTION 22.4 Electric Circuits: Ohm's Law and Joule's Law
6. Equation 22.6 is a statement of Ohm's Law, $V=IR$. Suppose you have two resistances, $R_1$ and $R_2$, in parallel with one another and you put a voltage $V$ across them. There will be a current of $I_1 = V/R_1$ through the first resistance $R_1$ and there will be a current of $I_2 = V/R_2$ through the second resistance. The total current in the circuit will then be equal to $I_1 + I_2$. What equivalent resistance $R$ do you need to hook across the voltage $V$ in place of $R_1$ and $R_2$ that will draw the same amount of current, namely $I_1 + I_2$? To answer this question let us set the potential difference equal to the potential difference across each separate parallel resistor, $V = R$ ($I_1 + I_2$) = $I_1R_1 = I_2R_2$. We can use the right sides of these equations to derive equations for the currents $I_1 = V/R_1$ and $I_2 = V/R_2$, so $V = R(V/R_1 + V/R_2)$;

$$1 = R(1/R_1 + 1/R_2) = R((R_2 + R_1) / R_1R_2)$$

$$R_1R_2 / (R_1 + R_2) = R$$ (1)

The equivalent resistance of two parallel resistors is the product of the resistances divided by the sum.
7. We can rewrite the Joule's Law expression of the power dissipated in a circuit by using Ohm's Law to replace the current in Equation 22.23 by the ratio V/R. Then
\[
\text{Power} = (V/R)^2 R = V^2/R
\]  
(2)
8. One experiment that can be used to relate the electrical energy to mechanical energy is to place a heating coil of wire in an insulated container that holds a known volume of water. Turn on the current and measure the potential difference, the current, the time, and the change in temperature of the water. You can then check to see if the VI\textit{t} product is equal to the volume times change in temperature times 4.19 joules/milliliter for the water.

**SECTION 22.5 Galvanometers**
9. The voltage across a 1000Ω galvanometer measuring a current of 10\(^{-9}\) A is 10\(^{-6}\) volts.
10. A perfect ammeter would have a resistance zero.
11. A perfect voltmeter would have an infinite resistance.
12. In the circuit where the ammeter is in series with the resistance, the voltmeter reading is the voltage drop across both the unknown resistance and the ammeter. But the reading of the ammeter is the correct reading for the current through the unknown resistance. The voltmeter reading must be reduced by an amount equal to the current times the ammeter resistance, then divided by the ammeter current reading to find the correct value of the unknown resistance.
\[
R = \frac{(V - I_A R_A)}{I_A}
\]  
(3)
In the circuit where the voltmeter is in parallel with the unknown resistance the ammeter reading is the current through both the voltmeter and the unknown resistance. So the current reading of the ammeter must be reduced by the amount of current in the voltmeter, then divided into the voltmeter reading to determine the correct value of the unknown resistance,
\[
R = \frac{V}{(I_A - V/R_V)}
\]  
(4)

**SECTION 22.7 The Wheatstone Bridge**
13. When the Wheatstone Bridge is balanced there must be no current through the null detector so points (a) and (b) of the circuit must be at the same potential. Thus the potential difference across \(R_1\) and \(R_2\) must be the same, i.e. \(V_1 = V_2\) or \(i_a R_1 = i_b R_2\). Similar reasoning can be used to deduce that \(i_a R_3 = i_b R_v\).
14. If we assume the current detector can act as an infinite resistor to measure and voltage difference between (a) and (b), then we can write down the voltage difference from the top of the emf to point (b) by two routes; through \(R_1\) and the current detector and through \(R_2\). The potential difference must be the same in each case
\[
i_R R_1 - V_{ab} = i_R R_2
\]
If we do not allow for any current flow through the detector then
\[
E = i_1 (R_1 + R_3) = i_2 (R_2 + R_v)
\]
so
\[
V_{ab} = i_1 R_1 - i_2 R_2
\]
\[
V_{ab} = ((E / R_1 + R_3) R_1) - ((E / R_2 + R_v) R_2)
\]
SECTION 22.10 The Piezoelectric Effect
15. A high input resistance device that draws very little current must be used with piezoelectric crystals.
16. The use of a periodic voltage to cause mechanical vibrations in a piezoelectric crystal is an example of a resonance phenomena. The frequency of the applied voltage must be the same as the natural frequency of the piezoelectric crystal.

SECTION 22.13 Electric Thresholds and Effects
17. Typical amounts of static electricity are not able to produce any appreciate current flow through humans so they are not usually dangerous.
18. The energy dissipated in an electric shock is proportional to the square of the current times the resistance times the time. The resistance of an object is inversely proportional to its area. The combination of these concepts, Equations 22.7 and 22.23, leads to the relationship
\[ \text{damage} \propto (\text{current}^2 / \text{area}) \times \text{time} \]

EXAMPLES

RESISTORS
1. (a) What is the equivalent resistance of \( n \) equal resistors in parallel? (b) Combine this result with the Least Common Multiple concept from mathematics to devise a technique for finding the equivalent resistance for unequal resistors in parallel. (c) Use the technique to find the resistor equivalent to 20W, 30W, and 50W in parallel.

What data are given?
The number of equal resistors = \( n \), an integer greater than zero.

What data are implied?
The circuit obeys Ohm's Law.

What physics principles are involved?
The conservation of electric charge and energy are used to derive the basic equations that are needed for this problem.

What equations are to be used?
We will use Ohm's Law to derive some other equations.
\[ V = IR \] (22.6)

Solutions
Assume there is a voltage drop \( V \) across each of these resistors of value \( R \). Then a current \( I \) flows through each branch of the circuit given by \( I = V / R \). But the current in the rest of the circuit must be equal to the sum of all the currents in parallel branches of the circuit, so
\[ I_{\text{total}} = I + I + ... I = nI \] since there are \( n \) branches, each carrying a current \( I \).
Thus we have a voltage of \( V \) and a current of \( nI \), what must the resistance be? The equivalent resistance \( R_{eq} \) must be

\[
R_{eq} = \frac{V}{nI} \quad \text{but} \quad \frac{V}{I} = R, \quad \text{so}
\]

\[
R_{eq} = \frac{R}{n} \quad \text{(5)}
\]

The equivalent resistance of \( n \) equal resistors in parallel is the resistance of one resistor divided by \( n \), the number of resistors in parallel.

(b) Now take a circuit of unequal resistors \( R_1, R_2 \) and \( R_3 \). We know from Equation (5) the resistor \( R_{eq} \), for example, can be considered the equivalent of some member, say \( n_{eq} \), of larger equal resistances, say \( R \), in parallel. Likewise \( R_2 \) can be equivalent to \( n_2 \) of the larger resistances \( R \) in parallel and \( R_3 \) equal to \( n_3 \) of the \( R \)'s in parallel. Then we can replace each of the unequal resistors by some numbers, \( n_{eq}, n_2 \) and \( n_3 \), respectively, of equal resistors and find the equivalent resistance of \( R_1, R_2 \) and \( R_3 \) in parallel using Equation (5).

\[
R_{eq} = \frac{R}{(n_1 + n_2 + n_3)} \quad \text{(6)}
\]

How can we find the value of \( R \) and of \( n_{eq}, n_2 \) and \( n_3 \)? Let us assume the \( R_1, R_2 \) and \( R_3 \) have no common factors, then the Least Common Multiple of the three is just their product.

\[
\text{L.C.M.} = R_1R_2R_3 \quad \text{(7)}
\]

If we choose the LCM of \( R_1, R_2 \) and \( R_3 \) for the value of \( R \) then we know that \( n_{eq}, n_2 \) and \( n_3 \) will all be integer numbers.

Choose \( R \) in ohms = \( R_1R_2R_3 \)

Then \( n_1 = \frac{R_1R_2R_3}{R_1} = R_2R_3; \quad n_2 = \frac{R_1R_2R_3}{R_2} = R_1R_3; \quad \text{and} \quad n_3 = \frac{R_1R_2R_3}{R_3} = R_1R_2 \)

Thus \( R_{eq} = \frac{R}{(n_1 + n_2 + n_3)} = \frac{R_1R_2R_3}{(R_1R_3 + R_1R_2 + R_2R_1)} \quad \text{(8)}
\]

(c) To see how this works let us consider the specific case of a 20\( \Omega \) resistor in parallel with a 30\( \Omega \) resistor in parallel with a 50\( \Omega \) resistor.

![Figure 22-2](image)

so \( n_{20} = \frac{300}{20} = 15; \quad n_{30} = \frac{300}{30} = 10; \quad n_{50} = \frac{300}{50} = 6 \)

So these three resistors in parallel are equivalent to 15+10+6, or 31, branches of 300\( \Omega \) resistors in parallel, thus

\[
R_{eq} = \frac{300}{31} = 9.7 \, \Omega
\]
Thinking about the answers

Look back over this procedure. Can you show that this result is equivalent to Equation (22.16)? Which technique do you prefer, the use of the sum of reciprocals, as in Eq. 22.16, or the use of the LCM as shown here? Take your pick, they give the same result.

OHM'S LAW

2. A 3Ω resistor and a 6Ω resistor are connected in parallel. This combination is connected in series with a 4Ω resistor; then the group of three resistors are connected in parallel with a 12Ω resistor. These four resistors are connected to the terminals of a 12V battery with an internal resistance of 2Ω. (a) Draw the circuit, labeling all the elements by their specified values. (b) Calculate the resistance of the combination of 4 resistors. (c) Calculate the current through each portion of the circuit. (d) Calculate the voltage drop across each portion of the circuit and show that energy is conserved in the circuit.

What data are given?

The answer to this question can also answer part (a) of this question, so let us draw a circuit diagram and label at the parts.

What data are implied?

It is assumed that all aspects of this circuit satisfy Ohm's Law.

What physics principles are involved?

The conservation of energy and electric charge are combined with Ohm's Law to derive all of the circuit equations to be used.

What equations are to be used?

\[
V_{\text{terminal}} = E - Ir
\]  \hspace{1cm} (22.8)

\[
V = IR
\]  \hspace{1cm} (22.6)

Series \[ \Sigma_{\text{total}} = \Sigma IR_i \]  \hspace{1cm} (22.9)

Series \[ R_{\text{eff}} = R_1 + R_2 + \ldots + R_n \]  \hspace{1cm} (22.10)

Parallel \[ I = i_1 + i_2 + i_3 \]  \hspace{1cm} (22.11)

Parallel \[ R_{\text{eff}}^{-1} = R_1^{-1} + R_2^{-1} + \ldots + R_n^{-1} \]  \hspace{1cm} (22.16)

Numerical Solutions

Rather than work out a complete algebraic solution for this problem before putting in numbers, let us work out algebraic solutions to each part of the problem, then obtain a numerical result and carry the numerical result from one part of the problem to another.
(b) To solve this portion of the problem we must begin with the 3Ω and 6Ω parallel combination so that we can combine that with the 4Ω resistor to find the equivalent resistance of those in parallel with the 12Ω resistor.

So the equivalent resistance of the top part of Figure 22-4 is 2Ω + 4Ω, or 6Ω.

The equivalent resistance of the four resistors is 4Ω.

(c) To calculate the current through each portion of the circuit we will find the current through the battery first and then trace its subdivision through a portion of the circuit. The 12V battery is connected to 4Ω of external resistance plus having an internal resistance of 2Ω, so

\[ I_{\text{battery}} = \frac{12V}{4\Omega + 2\Omega} = 2A \]

When the 2A current leaves the battery it meets an equivalent parallel circuit that contains 6Ω in parallel with 12Ω. The resistance (6Ω) is half as much in one branch so the current in that branch must be twice as much as in the other (12Ω) branch; thus

\[ I_{12} = \frac{2}{3}A; \quad I_{\text{top branch}} = \frac{4}{3}A \]

All of the \( \frac{4}{3}A \) will pass through the 4 resistor in the top branch. Note that the \( \frac{4}{3}A \) also comes to a parallel portion of the circuit where the ratio of resistances in the two branches is 2 to 1 (6Ω to 3Ω). The current through the 3W resistor will be twice the current in the 6Ω resistor. Thus

\[ I_3 = \frac{8}{9}A; \quad I_6 = \frac{4}{9}A \]

Of course, the current through the 2Ω resistance of the battery is the total of 2A. Notice how current is conserved at each branch point in the circuit.
(d) We can find the voltage drops across each portion of the circuit by using Ohm’s Law, \( V=IR \).
- \( V_3 = (8/9 \text{ A})(3) = 8/3 \text{ Volts} \)
- \( V_6 = (4/9 \text{ A})(6) = 8/3 \text{ Volts} \)
- \( V_4 = (4/3 \text{ A})(4) = 16/3 \text{ Volts} \)
- \( V_{12} = (2/3 \text{ A})(12) = 8 \text{ Volts} \)
- \( V_2 = (2\text{ A})(2) = 4 \text{ Volts} \)

**Conservation of energy**
- Potential across the 3\( \Omega \) and 6\( \Omega \) in parallel = 8/3 V.
- Potential across the 12\( \Omega \) and 3\( \Omega \) resistor combination = 8V.
- Potential across all resistances = 12V = Emf of the battery.

**POWER LOSS**
3. Find the power loss in each portion of the circuit in Problem 2 above.

**What data are given?**
- See Figure 22-4 and the answers to part (c) and (d) of question 2.

**What data are implied?**
- The circuit elements are ohmic.

**What physics principles are involved?**
- The use of Joule’s Law as discussed in Section 22.4.

**What equation is to be used?**
- Joule’s Law: \( P = VI = I^2R \)  
- (22.23)

**Solutions**
- \( P_3 = V_3I_3 = (8/3 \text{ V})(8/9 \text{ A}) = 64/27 \text{ W} = 2(10/27) \text{ W} = 2.4 \text{ W} \)
- \( P_6 = V_6I_6 = (8/3 \text{ V})(4/9 \text{ A}) = 32/27 \text{ W} = 1(5/27) \text{ W} = 1.2 \text{ W} \)
- \( P_4 = V_4I_4 = (16/3 \text{ V})(4/3 \text{ A}) = 64/9 \text{ W} = 7 1/9 \text{ W} = 7.1 \text{ W} \)
- \( P_{12} = V_{12}I_{12} = (8 \text{ V})(2/3 \text{ A}) = 16/3 \text{ W} = 5.3 \text{ W} \)
- \( P_2 = V_2I_2 = (4 \text{ V})(2 \text{ A}) = 8 \text{ W} \)

**Power Output of the Battery**
- \( EI = (12\text{ V})(2\text{ A}) = 24\text{ W} \)

**Is that equal to the total power loss of the complete system?**
- Power Loss = 2.4 + 1.2 + 7.1 + 5.3 + 8.0 = 24W

**Thinking about the answers**
The power supplied by the battery has to equal the power loss in the circuit for electrical energy to be conserved in this system. From where does the electrical energy come for this circuit?

Please note that if you were given a circuit and asked to find the power loss in each circuit element you would have to go through the procedures used in problem 2 first to find the current in each branch of the circuit, then use \( P=I^2R \) to find the power dissipated in each circuit element. A procedure which may be lengthy, but if done systematically will yield the correct results.
PRACTICE TEST

1. Briefly describe the difference between resistance (OHMS) and resistivity (OHM-meters). Give an example of how each term is used in a practical situation.

2. The circuit outlined by the diagram below was used to show the electrical flow in a DC circuit. The 1 OHM resistor is the internal resistance of the 75 volt battery.

![Circuit diagram]

Using this information, find the following:
A. The equivalent resistance of the circuit.
B. The current measured by the ammeter.
C. The voltage indicated by the voltmeter.
D. The "Terminal" voltage of the battery in the circuit.

3. Three heat lamps (120 volts, 200 watts) are to be connected to a 120 volt electrical outlet. Circuit 1 shows the bulbs connected in parallel.

![Circuit #1 diagram]

A. What is the voltage drop across each bulb in circuit 1?
B. What total current is required from the outlet where circuit 1 is connected?
C. Find the cost for operating the circuit for a time of 3 hours if the charge is $1.05 for each KWH.
4. An important application of electricity in human physiology is the ECG. A typical output signal is shown below.

A. What is this signal called and what does it control?
B. Briefly describe the electrical origins of the signal and what the function of each part of the wave serve.

ANSWERS:
1. OHM - Rating given to a part of an electrical circuit offering a resistance to the flow of charges. For a 1 volt potential which can produce a current of 1 Amp, the resistance is 1 OHM. OHM - Meter - A basic property of a material substance rather than an individual device. The property is independent of any size and/or shape. The rating can predict the ohmic resistance of a material after being formed to a certain size and shape.
2. 25W, 3 Amp, 36 volts, 72 volts
3. 120 volt - AC, 5 Amps, 9 cents ($.09)
4. Electrocardiogram (ECG) - Heart, the electric origin of the signal is found within the center of the brain. The function is to synchronize the heart’s pumping action: P (atrial changing dipole signal), QRS (Ventricular changing dipole), T (Ventricular dipole restoration).