Chapter 19
Wave Properties of Light

GOALS

When you have mastered the contents of this chapter, you will be able to achieve the following goals:

Definitions
Define each of the following terms, and use it in an operational definition:
- interference
- optical path length
- diffraction
- optical activity
- dispersion
- coherent
- polarization
- noncoherent
- birefringence

Application of Wave Properties
Explain the physical basis for:
- thin film colors
- polarimetry
- diffraction grating spectrometry
- holography
- resolving power of optical systems

Problems Involving Wave Properties
Solve problems involving interference, diffraction, and polarization.

Optical Activity
Design an experimental system capable of measuring the optical activity of a solution.

Lasers
Compare the laser with other light sources in terms of their optical characteristics.

PREREQUISITES
Before you begin this chapter you should have achieved the goals of Chapter 16, Traveling Waves, and Chapter 18, Optical Elements.
Chapter 19
Wave Properties of Light

OVERVIEW - Under most day-to-day conditions, we see light behave as though it were a wave. Primarily, these wave phenomena include interference, diffraction and polarization. In this chapter, you will be introduced to the quantitative aspects of these phenomena in addition to many applications of these phenomena.

SUGGESTED STUDY PROCEDURE - When you begin to study this chapter, read the following Chapter Goals carefully: Definitions, Applications of Wave Properties, Problems Involving Wave Properties, and Lasers. An expanded discussion of each of the terms listed under Definitions can be found in the next section of this Study Guide. Next, read text sections 19.1-19.5 and 19.7-19.11 and 19.14. Answers to questions posed in each section are discussed in this Study Guide chapter, section three.

At the end of the chapter, read the Chapter Summary and complete Summary Exercises 1-8. Next, do Algorithmic Problems 1-6 and Exercises and Problems 1-5, 7, and 17. For more work with the concepts of this chapter, see the Examples section of this Study Guide.

Now you should be prepared to attempt the Practice Test provided at the end of this chapter. If you have difficulties with any individual problem, refer to that particular part of the text or this chapter for assistance. This study procedure is outlined below.

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DEFINITIONS

INTERFERENCE - The superposition of two or more waves with constant phase differences produces interference.
   In a case when two waves meet in phase, the interference will be constructive. When the waves meet out-of-phase, the interference will be destructive.

DIFFRACTION - Occurs if a wave front encounters an object and there results a superposition of wave fronts.
   Diffraction is the bending of waves around objects. The most common example of diffraction is the spreading out (bending) of a wave which must pass through a narrow opening.

DISPERSION - A medium will produce a dispersion of waves if the wave velocity is a function of the frequency.
   As light passes through glass, different wavelengths (frequencies) are slowed by different amounts. Thus the wavelengths do not move the same distance in the exact same amount of time.

POLARIZATION - The process by which a light wave loses its random orientation of transverse propagation and only a single direction of transverse propagation is allowed.

BIREFRINGENCE - Property of crystals that have different velocity of light for different directions.
   This property allows double refraction of light.

OPTICAL PATH LENGTH - Equals thickness of sample multiplied by index of refraction for wavelength in the sample.
   More generally, this is the length from source to screen travelled by a light wave.

OPTICAL ACTIVITY - Phenomenon produced by certain materials that rotate the plane of polarization as light passes through.

COHERENT - Light emitted from a laser has each wave in phase with the other waves emitted.

NONCOHERENT - Light emitted from an ordinary incandescent bulb is randomly produced, thus the waves are not in phase.
SECTION 19.3 Interference

One condition necessary for interference between waves is that the frequency be the same for all the waves. Most common light sources emit a wide range of frequencies of light. So it may not be so obvious to the casual observer of natural events that interference occurs. The color patterns of an oil film on the surface of water and of some insect wings can be attributed to interference. Most light sources are polychromatic and noncoherent, so if the light waves are all present in the same region of space, a detector will measure the superposition of all the wave amplitudes. As shown in Figure 19.1, if two equal waves destructively interfere, then the resultant amplitude is zero. Such a case for two traveling waves would not hold true for all directions in space. The energy conservation concept applies to a total system, not just to some individual waves within the system. If the waves are added up over the whole system, then energy is conserved.

SECTION 19.4 Effective Optical Path Lengths

Since the index of refraction in a vacuum is by definition exactly one, then the error for not correcting for the index of refraction of air is equal to \((1.0003 - 1.0000)/1.00 = 33 \times 10^{-4} \) or 0.03%. That seems like a very small difference, right? But consider the number of complete wavelengths of yellow-green light that extend 1 cm in a vacuum compared to 1 cm of air. The wavelength of yellow-green light is, say 550.06 nm; then 18,180 wavelengths extend for 1.00000 centimeters in a vacuum, but that is 18,185 wavelengths in 1 cm of air, a difference of 5 wavelengths.

SECTION 19.5 Thin-Film Interference Patterns

The wings of insects can be studied using interference techniques. If you have available a monochromatic source of light whose wavelength you can vary, then you could study the various interference patterns for different wavelengths of light. What results might you expect?

SECTION 19.10 Polarization

We may construct almost any model for the emission of light from natural objects that we like, but as long as isotropic randomness of microscopic systems is preserved, as seems to be required by the second law of thermodynamics, unpolarized light would be most common from typical emission systems. One interesting way to study polarization effects in your daily environment is to tip your head 90° while wearing Polaroid sunglasses. With your head in its usual vertical position the plane of polarization of Polaroid sunglasses is vertical to screen out the intense horizontally-polarized reflected light. By tipping your head sideways you permit the horizontally polarized light to reach your eyes. Some strain patterns in automobile windows are observable with Polaroid sunglasses.
As shown in Figure 19.13, the fact that Rayleigh scattering is more pronounced for the short wavelength blue light than for red light can be used to explain the redness of sunsets and sunrises. The direct light falling on your eyes from the sun has more of its blue light scattered away so it appears more red than when directly overhead. This is a result of the increased distance the light travels through the earth’s atmosphere at sunrise or sunset, see the figure below.

The sunset you see almost daily,
Is attributed to Mister Rayleigh.
That redish hue,
Comes from blue,
That colors the sky, oh so gaily.
EXAMPLES

PROBLEMS INVOLVING WAVE PROPERTIES

1. What wavelengths of light will be most strongly reflected by an oil \( (n_{\text{oil}} = 1.40) \) film \( 2.80 \times 10^{-7} \) mm thick (a) floating on water \( (n_{\text{water}} = 1.33) \) or (b) on a glass plate with an index of refraction \( (n_{\text{glass}} = 1.50) \).

**What Data Are Given?**

The thickness of the oil film is \( 2.80 \times 10^{-7} \) mm. The indices of refraction of the three media, oil, water, and glass are given as 1.40, 1.33, and 1.50 respectively.

**What Data Are Implied?**

Assume the angle of incident of the light is 0°; i.e., the incoming light is perpendicular to the oil interfaces. The most strongly reflected waves are the ones which undergo constructive interference.

**What Physics Principles Are Involved?**

This problem uses the ideas of interference and phase shift upon reflection as discussed in Section 19.5 for constructive interference.

**What Equations Are to Be Used?**

Constructive Interference with no phase shift upon reflection at 2nd surface

\[
2nt = \left(\frac{m}{2}\right)\lambda \quad m = 1, 3, 5
\]

Constructive interference with a 180° phase shift upon reflection at both surfaces.

\[
2nt = \frac{\lambda}{2} (m + 1) 
\]

**Algebraic Solutions**

Note that in going from a region of lower index of refraction to higher index of refraction the reflection wave undergoes a phase change of 180° or \( \lambda / 2 \).

Case (a) air - oil - water - In this case there is a phase change at the air - oil surface only, thus \( 2nt = \left(\frac{m}{2}\right)\lambda \quad m = 1, 3, 5 \)

\[
\lambda_1 = 1.57 \times 10^{-6} \text{ m}; \quad \lambda_2 = 5.23 \times 10^{-7} \text{ m}; \quad \lambda_3 = 3.14 \times 10^{-7} \text{ m}; \quad \lambda_4 = 1570 \text{ nm}; \quad \lambda_5 = 523 \text{ nm}; \quad \lambda_6 = 314 \text{ nm}; \ldots
\]

Case (b) air - oil - glass - In this case there are phase changes at the air - oil and oil - glass surfaces, thus

\[
2nt = \frac{\lambda}{2}(m + 1) \quad m = 1, 3, 5, \ldots
\]

**Numerical Solutions**

(a) \( 2(1.40)(2.80 \times 10^{-7}) = \lambda_1 / 2 = (3\lambda_2) / 2 = (5\lambda_3) / 2 = \ldots \)

\[
\lambda_1 = 1.57 \times 10^{-6} \text{ m}; \\
\lambda_2 = 5.23 \times 10^{-7} \text{ m}; \\
\lambda_3 = 3.14 \times 10^{-7} \text{ m}; \\
\lambda_4 = 1570 \text{ nm}; \\
\lambda_5 = 523 \text{ nm}; \\
\lambda_6 = 314 \text{ nm}; \ldots
\]

(b) \( 2(1.40)(2.80 \times 10^{-7}) = \lambda_1 = 2\lambda_2 = 3\lambda_3 = \ldots \)

\[
\lambda_1 = 7.84 \times 10^{-7} \text{ m}; \quad \lambda_2 = 3.92 \times 10^{-7} \text{ m}; \quad \lambda_3 = 2.61 \times 10^{-7} \text{ m}; \quad \lambda_4 = 784 \text{ nm}; \quad \lambda_5 = 392 \text{ nm}; \quad \lambda_6 = 261 \text{ nm}; \ldots
\]
2. A diffraction grating has 1250 lines per centimeter (a) Over what range of angles will it spread the first order visible spectrum (350 nm to 700 nm)? (b) If you use this grating as a monochromator to produce light of 588 nm, what other wavelengths occur at the same angle?

**What Data Are Given?**
The distance between the slits = \( d = 1 \text{ cm} / 1250 = (10^{-2} \text{ m}) / 1.25 \times 10^3 = 8.00 \times 10^{-6} \text{ m} \) wavelength's range; 350 nm ≤ \( \lambda \) ≤ 700 nm

**What Data Are Implied?**
The problem asks you to calculate the angles of constructive interference from a diffraction grating, see Figure 19.6, assuming normal (90°) incident light.

**What Physics Principles Are Involved?**
The problem uses interference from a diffraction grating as discussed in Section 19.8.

**What Equation is to be Used?**
\[ d \sin \theta = m \lambda; \quad m = 1, 2, \ldots \] (19.3)

**Solutions**
(a) The smallest angle will occur for the smallest wavelength; for first order \( m = 1 \)
\[ \sin \theta_{\text{min}} = \lambda / d = 350 \text{ nm} / 8.00 \times 10^{-6} \text{ m} = (3.5 \times 10^{-7}) / (8.0 \times 10^{-6} \text{ m}) = 4.38 \times 10^{-2} \]
The largest angle will occur for the largest wavelength
\[ \sin \theta_{\text{max}} = 700 \text{ nm} / 8.00 \times 10^{-6} \text{ m} = (7.0 \times 10^{-7}) / (8.0 \times 10^{-6} \text{ m}) = 8.75 \times 10^{-2} \]
\[ \theta = 5.0^\circ \]
So the first order visible spectrum is spread between the angles of 2.5° and 5.0° with respect to the incident angle. Compare these angles with the example on p. 439. What accounts for the differences?
(b) From Equation 19.3 you can see that the angle \( \theta \) will be the same for all constant products for a given grating. Hence the first order 588 nm light will coincide with second order 294 nm; third order 196 nm; fourth order 147 nm, sixth order 98 nm, seventh order 84 nm; etc. None of these wavelengths are visible, other than 588 nm. However, if you are using a grating monochromator with transducers that have a wider spectral response than the human eye, care must be taken to avoid multiple orders.

3. A lens is used to convert the light emitted from a light bulb into a plane wave of light traveling in the positive x-direction. Three polarizers are arranged along the x-axis with their axes of polarization along the y-axis, 30° from the y-axis, and 90° from the y-axis, as shown below. Assume that the intensity before passing through the first polarizer is \( I_0 \). Find the intensity, amplitude, and direction of polarization at (a) point A, (b) point B, and (c) point C.
What Data Are Given?
The initial intensity $I_o$ and the angles of polarization of three polarizers.

What Data Are Implied?
It is to be assumed that the light at the beginning is unpolarized of intensity $I_o$, associated with an electromagnetic wave of amplitude $E_o$.

What Physics Principles Are Involved?
The concepts of polarization as related to a transverse wave of amplitude $E_o$ are needed for this problem. See Section 19.10 and Figure 19.11.

What Equations Are to be Used?
\begin{align*}
E &= E_o \cos \theta \\
I \propto E^2 &= E_o^2 \cos^2 \theta \\
I &= I_o \cos^2 \theta
\end{align*}

Algebraic Solutions
(a) The amplitude of the wave is unchanged by an ideal polarizer; so $E_A = E_o$. Since the light at the beginning is completely unpolarized we can resolve it into two components that are right angles to one another. Then half of the intensity at Point A is one-half $I_o$, $1/2 I_o$. The direction of polarization is parallel to the y-axis.

(b) The light now polarized along the y-axis of amplitude $E_o$ and intensity $I_o/2$, comes to the second polarizer oriented $30^\circ$ from the y-axis. The amplitude after passing through the second polarizer is $E_o \cos \theta$ or $E_o \cos 30^\circ$ for this case. The intensity is reduced by a factor of $\cos^2 \theta$ or $\cos^2 30^\circ$ for this case.

At Point B:
\begin{align*}
E_B &= E_A \cos 30^\circ = 0.87 E_o; \\
I_B &= I_A \cos^2 30^\circ = 0.75 I_A = 0.38 I_o
\end{align*}
The angle of polarization is $30^\circ$ from the x-axis.

(c) The light is now polarized at an angle of $30^\circ$ to the y-axis, with amplitude 0.87 $E_o$ and intensity 0.75 $I_o$. It now comes to a polarizer at an angle of $60^\circ$ to its plane of polarization. So the amplitude at point C is $E_B \cos \theta_{BC}$ or $E_B \cos 60^\circ$ or 0.87 $E_o \cos 60^\circ$. The intensity at point C is $I_C = I_B \cos^2 \theta_{BC} = I_B \cos^2 60^\circ$.

At Point C:
\begin{align*}
E &= 0.87 E_o \cos 60^\circ = 0.44 E_o \\
I_C &= I_B \cos^2 60^\circ = 0.38 I_o (0.25) = 0.10 I_o
\end{align*}
The polarization is parallel to the z-axis.

Thinking About the Answer
Notice that the first and third polarizers are at right angles to one another. If they were the only polarizers in the system NO light would pass through the system. The insertion of a third polarizer between the two crossed polarizers allows some light to pass through the system. In this case, it is at an angle of $30^\circ$ with respect to the first polarizer and 10% of the initial light gets through the system. Is that the maximum amount you can get through such a system? What is the best angle for the middle polarizer in order to obtain the maximum intensity through the system?

When dealing with Rayleigh’s criteria,
The students displayed mass hysteria.
For tests they had cram’d a,
One point two times lambda,
And such terms as hypermetropia.
PRACTICE TEST

1. A thin air wedge is formed from two microscope slides and a thin piece of masking tape. The wedge is illuminated with green light.

   ![Diagram of air wedge](image)

Will the wedge at point A appear dark (black) or bright (green)? Explain your answer by treating the "wedge" as a thin film.

2. If one looks at a light bulb filament through a piece of sheer curtains the pattern seen is that of a diffraction grating. The first order pattern is centered at 0.01 radians with respect to the central maximum. If a wavelength of 500 nm is used, calculate the thread spacing in the sheer curtains.

3. How does the light from a laser source differ from the light emitted by an ordinary source, like an incandescent bulb?

ANSWERS:

1. At A, wedge appears dark. Light entering wedge undergoes 180° phase change and is reflected. At bottom surface, reflected light undergoes no phase change. Therefore, since the optical path approaches zero, the two reflections will be 180° out of phase and produce destructive interference.

2. $5 \times 10^{-3}$ cm

3. Light from ordinary sources is incoherent, or not produced in a constant phase relationship. Emitted light is randomly produced in an incandescent bulb. Normal light sources (white) contain many wavelengths which taken together produce a nearly white color. Laser light is both monochromatic (single wavelength or color) and coherent (produced with a single phase relationship).