Chapter 18
Optical Elements

GOALS
When you have mastered the content of this chapter, you will be able to achieve the following goals:

Definitions
Define each of the following terms and use it in an operational definition:
light ray
optical axis
object distance
converging optical elements
image distance
diverging optical elements
index of refraction
real image
reflection coefficient
virtual image
internal reflection
magnification
focal point
aberrations - chromatic and spherical

Ray Diagrams
Draw ray diagrams for some common optical systems.

Lens and Mirror Equations
Apply the basic equations for lenses and mirrors to optical systems with one or two components.

Optical Devices
Explain, using physical principles, the operation of a reading glass, camera, microscope, and fiber optics.

PREREQUISITES
Before beginning this chapter you should have achieved the goals of Chapter 16, Travelling Waves.
Chapter 18
Optical Elements

OVERVIEW
Light travels rapidly through free space in a straight line. When light rays are deviated from this straight line motion, either reflection or refraction is usually involved. In this chapter you will be introduced to the reflection properties of plane and curved mirrors plus the refraction properties of single thin lenses and combinations of lenses. The image producing properties of each of these phenomena will be outlined with application examples.

SUGGESTED STUDY PROCEDURE
As you begin to study this chapter, be familiar with these Chapter Goals: Definitions, Ray Diagrams, Lens and Mirror Equations, and Optical Devices. Remember to look at the next section of this Study Guide for an expanded discussion of each of the terms listed under Definitions. Next, read chapter sections 18.1-18.9. Remember that answers to questions asked in the text are answered in the second section of this Study Guide Chapter. As you read, pay special attention to the many figures which illustrate the properties of reflection, refraction, and image formation. Also, be sure to practice the rules given on page 408 and 412 for drawing ray diagrams for curved mirrors and thin lenses, respectively. Although Section 18.9 is short, the information presented about cameras is important. See the examples section of this Study Guide for additional information.

At the end of the chapter, read the Chapter Summary and complete Summary Exercises 1-16. Now do Algorithmic Problems 1-4 and complete Exercises and Problems 1, 8, 9, 10 and 12. For additional practice with the concepts presented in this chapter, work through the additional problems provided in the Examples section of this Study Guide chapter. Now you should be prepared to attempt the Practice Test given at the end of this Study Guide chapter. If you are unsuccessful with any part, refer to that portion of the text. This study procedure is outlined below.

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<td></td>
<td>18.12</td>
<td></td>
<td></td>
<td>14,15</td>
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**DEFINITIONS**

**LIGHT RAYS** - Radii of spherical waves. Imaginary lines drawn in the direction of the light wave propagation.
These rays are of special help when drawing diagrams for locating images formed by concave mirrors and convex lenses.

**OBJECT DISTANCE** - The axial distance from the center of a lens or mirror to the object position.

**IMAGE DISTANCE** - The axial distance from the center of a lens or mirror to the image position.

**INDEX OF REFRACTION** - Ratio of the velocity of light in a vacuum to the velocity of light in the refractive material.
The index number gives a rating of a transparent material's ability to bend light as it transverses from outside to inside the material. Diamond has a high ability to bend light \( n = 2.4 \) whereas water has less ability to bend light \( n = 1.33 \).

**REFLECTION COEFFICIENT OF LIGHT** - For normal incidence, the fraction of light intensity reflected.
The reflection coefficient depends upon the relative comparison of the index of refractions of the materials on either side of the reflection boundary.

**INTERNAL REFLECTION** (total) - When angle of incidence is equal to or greater than critical angle \( q_c \), light is totally reflected back into incident medium.
This reflection occurs only when light traveling in a medium of high index of refraction is incident on a boundary where a medium of lower index of refraction exists.

**FOCAL POINT** - The location on the axis of a lens or mirror where parallel light converges to a point.
In the special cases of the convex mirror and the concave lens, the focal points are located at a point on the axis where parallel rays seem to the observer to have originated.

**OPTICAL AXIS** - The line of symmetry through the center of a lens or mirror.
The focal point and the center of curvature point are both located on the optical axis.

**CONVERGING OPTICAL ELEMENTS** - Optical elements which converge parallel rays of light to a focus and can produce a real image. Both the concave mirror and the double convex lens are of this type.

**DIVERGING OPTICAL ELEMENTS** - Optical elements which diverge parallel rays of light and form virtual images. Both the convex mirror and the double concave lens are examples of this type.

**REAL IMAGE** - An image formed by a mirror or a lens which can be displayed on a screen. Real light rays actually converge to form this image.

**VIRTUAL IMAGE** - An image formed by either a mirror or a lens which can be seen but cannot be displayed on a screen. These images seem to exist but real light rays do not converge to produce them.
MAGNIFICATION - The ratio of image size to object size. The ratio of the angle subtended by the image with lens to an angle subtended by the object at the near point.

The magnification value may be used to predict how large (or small) an image will appear in relation to the original object.

CHROMATIC ABERRATION - The effect produced by the dependence of focal length on the frequency (color) of light.

This effect is mainly of concern with lenses. As light of a different color (wavelength) is bent by the lens, different colors are bent by different amounts. Thus the focal length will be different for different colors.

SPHERICAL ABERRATION - The effect resulting in different focal lengths for off-axis rays when incident on spherical lenses or mirror surfaces.

Even for a lens which is perfectly spherical, a true focal point will be difficult to produce. Since rays bent by the outer edges of the lens or mirror are bent more than rays along the optical axis, these rays will be influenced more.

ANSWERS TO QUESTIONS FOUND IN THE TEXT

SECTION 18.2 Straight-Line Wave Propagation and Ray Diagrams
Let us draw your attention to the relative wavelengths of sound waves and light waves. Typical audible sounds have wavelength of the order of 1 meter. The wavelength of typical visible light is $5 \times 10^{-7}$ m. So sound waves are about 10 million times as long as light waves. Consider the properties of waves discussed in Chapter 16 and propose various phenomena that might occur because of the great difference in wavelength between sound and light.

SECTION 18.6 Reflection from Spherical Mirrors
You will notice that it is only for the case shown in Figure 18.11 (a) do the actual rays pass through the image and the image is located on the object side of the mirror. Hence, only the image in (a) is a real image and can be shown on a screen. For both (b) and (c) of Figure 18.11, the images are behind the mirror. The rays must be extended behind the mirror to intersect at the location of the image. A virtual image cannot be shown on a screen, but must be viewed through an optical system, such as the human eye, capable of constructing an image from the rays that appear to come from the virtual image.

SECTION 18.7 Lenses
The power of a lens to refract and focus rays depends upon the ratio of the index of refraction of the lens to the index of refraction of the surrounding medium. If the index of refraction of the media is increased then the focusing power of the lens is decreased. The focal length of the lens is lengthened.

SECTION 18.8 Magnifier or Reading Glass
Examples:

1. You can use Equation 18.14 for calculating the magnification of a typical reading glass for a person with a near point 25 cm from the person's eye.

   \[ M = 1 + \frac{25 \text{ (cm)}}{f \text{ (cm)}} = 1 + \frac{25}{10} = 3.5 \]

   How is the magnification changed if your near point is 15 cm?
2. You can use Equation 18.14 to find the focal length of a lens if you know the magnification.

\[ M' = 1 + \frac{25}{f}; \text{ so } f = \frac{25}{(M-1)} = \frac{25}{7} = 3.6 \]

An experimental way to determine the focal length of a lens is to measure the distance from the lens to a real image of a distance object. Use Equation 18.13 to justify this experimental procedure. How distant must the object be to determine the focal length within ±10% using this method?

SECTION 18.9 Cameras
The total energy that enters a camera lens and falls on the film is proportional to the product of the lens area, the light intensity and the time;

\[ \text{Energy} = \text{Area} \times \text{Intensity} \times \text{Time} \]

\[ f \text{- number} = \frac{\text{focal length}}{\text{diameter of lens aperture}} \]

Area = \( \frac{\pi}{4} \) (diameter)\(^2\) =\( \frac{\pi}{4} \times (f.l./(f - \text{no.}))^2 \)

The area is proportional to the inverse square of the f-number.

<table>
<thead>
<tr>
<th>f - no.</th>
<th>1.4</th>
<th>2.0</th>
<th>2.8</th>
<th>4</th>
<th>5.6</th>
<th>8</th>
<th>11</th>
<th>16</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(^2) x inverse square of f-no.</td>
<td>510</td>
<td>250</td>
<td>130</td>
<td>63</td>
<td>32</td>
<td>16</td>
<td>8.3</td>
<td>3.9</td>
<td>2.1</td>
</tr>
</tbody>
</table>

To keep the total energy constant the product of the area and the exposure time must be held constant.

\[ \text{Time} = \text{constant} / (\text{inverse sq. of the f-no.}) = \text{constant} \times (\text{f-no.})^2 \]

If an exposure time of 1/100 of a second is correct for an f-number of 1.4, then the approximate exposure table is as below

<table>
<thead>
<tr>
<th>f-number</th>
<th>1.4</th>
<th>2.0</th>
<th>2.8</th>
<th>4</th>
<th>5.6</th>
<th>8</th>
<th>11</th>
<th>16</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>exposure</td>
<td>.01</td>
<td>.02</td>
<td>.04</td>
<td>.08</td>
<td>.16</td>
<td>.32</td>
<td>.61</td>
<td>1.26</td>
<td>2.42</td>
</tr>
</tbody>
</table>

Hence, the exposure time doubles each f-number, or the energy per unit area is decreased by 50% each time.

SECTION 18.11 The Compound Microscope
The image you see in a microscope is enlarged, inverted, and virtual. You cannot show a microscope image on a screen without adding an additional lens to make a real image on a screen. In order to photograph a microscope image you need to use a camera with a focusing lens on it.

EXAMPLES

LENS AND MIRROR EQUATIONS
1. An object is placed a distance equal to twice the focal length in front of (a) a diverging spherical mirror and (b) a diverging spherical lens. Locate the image in each case. Characterize the image and compute the magnification.

What Data Are Given?
Two different optical elements: a diverging (convex) mirror and a diverging (concave) lens. According to the sign convention given in the text, both of them will have negative focal lengths, say - f.
What Data Are Implied?
The conditions are assumed to be appropriate for Equations (18.8) and (18.13) to be used.

What Physics Principles Are Involved?
The basic physics of geometric optics was used to derive the equations for this problem.

What Equations Are to Be Used?

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad (18.8 \text{ or } 18.13)
\]
\[
m = -\frac{q}{p} \quad (18.6 \text{ or } 18.9)
\]

Algebraic Solution

\[
p = 2f; \text{ then if } f \text{ is a positive number}
\]
\[
\frac{1}{2f} + \frac{1}{q} = \frac{1}{-f}; \quad \frac{1}{q} = -\frac{1}{f} - \frac{1}{2f} = (-2 -1)/2f
\]
\[
q = -2f/3; \text{ so } q \text{ is virtual and less than } f.
\]
\[
m = -(-2f/3)/2f = 1/3; \text{ positive } m \text{ indicates an upright, virtual image}
\]

Ray Diagram Solutions

Thinking About the Answer
Even though these two different optical elements give the same results, notice the difference in the location of the image. In one case the image is behind the optical element and in the other case the image is in front of the optical element. If you have a chance in a laboratory experiment, compare these two optical elements by using them to view different objects.

2. Two thin lenses each of focal length +0.20 m are 0.20 m apart. An object is located 0.40 m left of the left lens. Find the magnification and location of the final image using a ray diagram and appropriate equations.

What Data Are Given?
The location of the object = 0.40 m. The focal lengths of the two lens = ± 0.20 m.
The distance between the two lens = 0.20 m.

What Data Are Implied?
The lens equations can be used so the object is appropriate to the sizes of the lenses.

What Physics Principles Are Involved?
The basic principles of geometric optics are used as given in Equations 18.9 and 18.13.
What Equations Are to be Used?

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]  \hspace{1cm} (18.13)
\[ m = -\frac{q}{p} \]  \hspace{1cm} (18.9)

**Algebraic Solution**

let \( f_1 \) = focal length of left lens, \( f_2 \) = focal length of right lens, \( d \) = distance between the lenses, and \( p_i \) = object distance from left lens

\[ \frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1}; \quad q_1 = \left(\frac{f_1 p_1}{p_1 - f_1}\right) \]  \hspace{1cm} (1)

where \( q_1 \) is the location of the first image.

\[ p_2 = \text{object distance with respect to the right lens} = d - q \]

then \( q_2 = \frac{(f_2 p_2)}{(p_2 - f_2)} = \frac{f_2(d - d_1)}{(d - q_1 - f_2)} \)  \hspace{1cm} (2)

Total magnification = \( -\frac{q_1}{p_1} - \frac{q_2}{p_2} = \left(\frac{f_1}{(p_1 - f_1)}\right) \left(\frac{f_2}{(d - q_1 - f_2)}\right) \)  \hspace{1cm} (3)

**Numerical Solution**

\[ q_1 = \frac{(0.20)(0.40)}{(0.40 - 0.20)} = 0.40 \text{ m} \]

thus \( p_2 = 0.20 \text{ m} - 0.40 \text{ m} = -0.20 \text{ m} \)

final image = \( q_2 = \frac{(0.20)(-0.20)}{(-0.20 - 0.20)} = -0.04/-.40 = 0.1 \text{ m to the right of the right lens} \)

magnification = \( (0.20/\ (0.40 - 0.20))\) \((0.20/\ (-0.20 - 0.20))\)
magnification = \( 1(0.20/\ -0.40) = -1/2 \)

**Ray Diagram**

inverted; real:
reduced image
Thinking About the Answer

For multiple lens or mirror problem it is important to pay careful attention to the sign convention used to determine which terms are treated as positive and negative. The use of a ray diagram to check your answer is a helpful technique.

Oh, the mirror in Tom's new boutique,
Can transform even Zorba the Greek!
From the fat to the thin,
From the neck to the shin,
It can make you and me look so sleek!
PRACTICE TEST

1. An object (represented by the arrow) is placed on the axis in front of a concave mirror as shown below. The focal point is 20 cm. from the mirror and the object is placed at 10 cm.

   ![Diagram of concave mirror with object and focal point marked]

   a. Calculate the image distance.
   b. Draw the principal ray diagram to locate the image formed by the lens.
   c. Describe the image: Is it real or virtual, upright or inverted? Calculate the magnification of this mirror system.

2. A classroom slide projector contains a converging lens of focal length 10.0 cm. It projects an image on a screen that is 2.50 m from the lens.
   a. What is the distance between the slide and the lens?
   b. What is the magnification of the image?
   c. Calculate the width of the image of a slide 35 mm wide formed on the screen.

3. Suppose you have a camera with a 10 cm. focal length.
   a. What is the diameter of the lens aperture for an F number of 2.8 (f/2.8 setting)?
   b. With this lens the proper exposure time for a given scene is 1/100 sec. at the setting f/2.8. What would be the exposure time for the same scene at a f/5.6 setting?

   ![Diagram of camera lens with focal length marked]

ANSWERS:
1. -20 cm; virtual, upright, larger, m = +2 (See Figure)
2. 10.4 cm, 24, 84 cm
3. 3.6 cm, .04 s